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Three essays on long memory tests for persistence in volatility and structural vector autoregression modeling of real exchange rates

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Three essays on long memory tests for persistence in volatility and structural vector autoregression modeling of real exchange rates

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ABSTRACT

In the first chapter the performance of two of the long memory tests, the Modified Rescaled Range Test and Geweke and Porter-Hudak Test for persistence in small samples is examined using Monte-Carlo methods. Some possible candidates for persistence in volatility are Autoregressive Conditional Heteroskedasticity (ARCH), Markov Regime Switching ARCH, and long memory. The long memory series are simulated through a Semi-Markov process with Pareto waiting times and lognormal realizations. The persistence in volatility arising from transition waiting probabilities for a Markov Regime Switching process, and from the tail index of the waiting time distribution for the Semi-Markov process is established through simulations with different parameter values. There is evidence that persistence in a regime switching process is closely linked to state transition probabilities and waiting times.

The second chapter re-examines what structural vector autoregressive modeling of real exchange rates with differenced variables tells us about interesting macroeconomic questions. Using quarterly data from G-7 countries in the post Bretton-Woods period, the evidence suggests that shock identification is not an easy process in a Blanchard and Quah decomposition framework with long run restrictions. Confidence bands do not find significant impulse responses and the signs of the estimated impulse responses are very sensitive to the lag selection criteria adopted. Possible cointegration effects seem to be the main driving force behind the unsatisfactory performance of the structural approach.

Chapter three extends the structural vector autoregression model by incorporating cointegration effects. Using the method of Warne (1993), in a simple four-variable VAR characterized by cointegration, the response of real exchange rates to various economic shocks are investigated with economically plausible long-run restrictions. The long-run relations and driving stochastic trends of the real exchange rate between United States and other G-7 countries are analyzed in a structural cointegrated framework. Productivity shocks depreciate the real exchange rate and the perverse sign effect of supply shock is corrected for most countries in the sample. More significant impulse responses are observed through confidence intervals. The structural vector error correction decompositions are also found to be not robust to estimating with different lag lengths owing to additional cointegration effects.

GENERAL INTRODUCTION

Introduction:

Returns in financial markets display persistent features of volatility suggestive of a time-dependent variance process. Various approaches have been advanced to model time-dependent variances. The basic building block of time-varying variance processes is the autoregressive conditionally heteroskedastic (ARCH) model with an AR(1) process in variance. However. ARCH is a short memory process and has failed to explain the overly persistent, or 'long memory' features in observed in high frequency financial data. One way to add additional persistence to simple ARCH is to introduce more than one regime driven by different autoregressive parameters. An ARCH process with two separate high and low volatility regimes can be characterized by Markov Regime Switching ARCH (MRSARCH). The number of countable regimes can be increased by introducing a Semi-Markov process where the process wanders across different regimes with the jump times from one regime to another determined by a waiting time distribution. Such a process exhibits long memory features reminiscent of financial market data. The first chapter takes a close look at how the persistence of a process is related to transition probabilities or waiting times, and whether the commonly used long memory tests, Modified Rescaled Range (MRR) test, and Geweke-Porter-Hudak test (GPH) can detect long memory behaviour.

Chapters 2 and 3 are closely related in the sense that they both examine the shock identification in VAR systems with and without the effect of cointegration among the used variables in the VAR system. Real exchange rates is one of the most commonly investigated variables in international finance literature and economists have shown great interest in identifying the sources of real exchange rate fluctuations. The results of such an endeavor may prove to be quite beneficial, in particular for understanding deviations from purchasing power parity and to appraise competing equilibrium theories about the real exchange rate. Structural Vector Autoregressions (SVARs) have found common use for this enterprise. Starting with simple models identified by contemporaneous restrictions on various shocks in the VAR system, economists have modified the VARs with long-run restrictions to arrive at more plausible identification schemes. SVAR analysis has often been burdened by cointegration relationships and such co-movements have been either ignored or not modeled in the literature. This has created complications for standard SVAR analysis and the last two chapters dwell on the identified problems and offer a solution through a structural vector error correction model.

Organization of Dissertation

The dissertation has two distinct objectives and three separate chapters. The first chapter carries out a Monte-Carlo simulation analysis to establish the relationship between regime switching behaviour and long memory features using two long memory tests. My objective is to see how persistence can be created in a regime switching process and whether that can be identified through tests both in time and frequency domain. The remaining two chapters do not share any common features with the first chapter of the dissertation. Chapters two and three set out with the objective of identifying problems associated with standard applications of structural vector autoregressions (SVARs) in shock identification in VAR setting with real exchange rates. My emphasis is on the floating exchange rate period and the sample I use is very similar to the data set in Clarida and Gali (1995, henceforth CG). CG were the first to use long-run restrictions in a structural VAR system. Later disciples of SVAR decompositions have followed through their steps but commonly used differenced variables in the VAR systems. In the second chapter I take the CG model to a slightly different data with an extended set of countries and identify serious problems with lag selection and possible cointegration effects. In the final chapter of the dissertation and I propose a structural vector error correction model that incorporates cointegration and use the levels of the variables in the system. I compare results from both approaches and argue for the additional merits of using a more advanced technique for structural shock identification for real exchange rates for the G-7 countries.

CHAPTER 1
REGIME SWITCHING AND PERSISTENCE IN VOLATILITY: A MONTE-CARLO INVESTIGATION

1. Introduction

In conventional econometric models, the variance of the error term is assumed to be constant. However, many economic time series and asset values exhibit phases of relative tranquility followed by periods of high volatility. For a series exhibiting volatility, the unconditional variance may be constant even though the variance during some periods is unusually high giving rise to conditional heteroskedasticity. Financial variables such as stock returns can be quite easily modeled by martingale difference sequences, as such models are justified by efficient financial markets. A martingale difference sequence is characterized by constant unconditional variance, and is serially uncorrelated. Even though financial series appear to be uncorrelated, they cannot be assumed independent and identically distributed. Melino and Turnbull (1990) and Tauchen and Pitts (1983) have shown that the variance of such series tends to be time dependent, in the sense that large and small values appear in clusters, suggestive of a time-varying variance process.

Several different approaches have been proposed to model time-dependent variances. Engle (1982) uses an autoregressive conditionally heteroskedastic model. The conditional variance of a series at time t depends on its past values through an autoregressive process. Bollerslev (1986) generalized Engle's model to include autoregressive moving average (ARMA) processes, as "Generalized AutoRegressive Conditional Heteroskedasdicity" (GARCH).

GARCH applications involving high frequency financial data have indicated the presence of a unit root in the univariate representation for the volatility. (Lamoureux, and Lastrapes, 1990) Financial market volatility displays persistent features as observed by very slowly decaying autocorrelations for absolute and squared returns. GARCH is a short memory model, and thus one way to mimic such a strong observed persistence is by using or approximating a unit root. Engle and Bollerslev (1986) introduced the integrated GARCH (I-GARCH) process, in which shocks to variance do not decay over time and the current information remains important for the forecasts of the conditional variances for all future horizons. Integration in variance is analogous to a unit root in the mean of a stochastic process. Lamoureux and Lastrapes (1990) pointed out that the potential problem with I-GARCH is that it lacks theoretical motivation. However, some researchers have not hastened to suggest a unit root in the variance structure. In order to avoid a possible criticism on the assumption of a drift in variance, it has been suggested that the variance can be characterized by a long memory process. Taylor (1986) realized that the absolute values of stock returns tended to have very slowly decaying autocorrelations. Baillie, Bollerslev and Mikkelsen (1996) considered a long memory process in the variance known as "Fractionally Integrated Generalized Autoregressive Conditional Heteroskedasticity" (FIGARCH). This process implies a slow hyperbolic rate of decay for autocorrelations of squared innovations and persistent impulse response weights. Therefore, long memory stands as a possible candidate to explain persistence in volatility of a series.

One needs to be cautious about taking general evidence of I-GARCH or strong persistence at face value. Regime switching may give rise to persistence that is observationally equivalent to a unit root. The strong serial dependence manifested in the evolution of volatility may be an artifact of an exogenous driving variable. Lamoureux and Lastrapes (1990) investigated the possibility that the appearance of high persistence in variance in daily stock return data is due to time-varying GARCH parameters. For example I-GARCH might be due to instability in the unconditional variance. Such a possibility was confirmed by Lastrapes (1989), who showed that exchange-rate volatility, as measured by ARCH, depends on monetary policy regimes. Different targeting policies on the part of Federal Reserve (FED) incorporates changing volatilities to the evolution of exchange rate and consequently to the stock market. Such shifts bias upward GARCH estimates of persistence in variance.

This paper will investigate such a possibility using nondeterministic shifts as opposed to Lamoureux and Lastrapes (henceforth LL) through a two-state Markov chain. LL found in their 1990 study that allowing for the presence of deterministic shifts in the conditional variance intercept in GARCH produces substantially lower estimates of the persistence parameters. Hamilton (1988) tried to model the effects of dramatic shocks and political events in financial asset prices through a Markov Switching Regime. Hamilton also documented the success of the switching regime model for short-term interest rates.

The purpose of this study is to try to establish the size and the power of the GPH and MRR tests on three different data generating processes which include the possibility of long memory in the observed persistence in variance. Monte Carlo simulations will generate standard ARCH processes, ARCH processes with regime switching through a Markov Chain (henceforth referred to as MRSARCH) and long memory series as possible candidates for the persistence in variance. Cheung (1993) provides tables regarding the size of MRR and GPH tests for long memory and first order ARCH processes with various autoregressive coefficients. Cheung's study comprises of long memory processes with a support on the real line, whereas this paper deals only with long memory processes with a positive support in their distribution. This study will demonstrate the size of GPH and MRR tests for switching regimes. Different parameterizations of MRSARCH will provide us with valuable information as to the relationship of the persistence in volatility to state transition probabilities. The long memory process will be generated through regime switching following Liu (2000). Instead of modeling regime switching through a discrete state Markov chain as in Hamilton and Susmel (1994), we model regime switching as a transition across i.i.d. regimes with the duration distribution for each regime defined as a heavy-tailed Pareto distribution. The simulations confirm Liu (2000) that the long memory behavior is closely related to the tail index of the duration distribution and also that strong persistence to stay in a given state in a MRSARCH process leads to more rejection of MRR and GPH tests.

The paper is organized as follows: Section 2 surveys the literature for long memory and provides a theoretical background both in time and frequency domain. Section 3 deals with the general features of standard ARCH and Markov Regime Switching ARCH and establishes how a switching process results in long memory and thus persistence in volatility. Section 4 lays out the algorithm to generate the three different processes; ARCH, MRSARCH, and long memory. Section 5 lays out the procedure to apply MRR and GPH tests. Section 6 presents the simulation results and also includes figures. Section 7 concludes. The Appendix elaborates on the derivations used in simulations and throughout the sections.

2. Long memory in Time and Frequency Domain

Long memory time series are mainly characterized by slowly decaying autocorrelations. In the usual autoregressive moving average (ARMA) representation of a time series the autocorrelations die exponentially, whereas long memory processes display persistence in their autocovariance functions exhibiting a hyperbolic decay. Long memory, defined by properties of the correlogram and spectrum, is concerned essentially with the linear properties of a process.

There are a number of other processes that can also mimic long memory series, including generalized fractionally integrated models arising from aggregation, time-changing coefficient models, and possibly nonlinear models as discussed by Granger and Hallman (1991) and Ermini and Granger (1993).

Long memory in frequency domain can also be characterized with an infinite spectrum at zero frequency. The failure of standard autoregressive integrated moving average (ARIMA) models to represent the spectral density at low frequencies adequately suggests that many-step-ahead forecasts obtained using those models could be inferior to those produced by models that permit unbounded spectral densities at frequency zero and autocorrelation functions that do not decay exponentially. The long memory model proposed independently by Granger and Joyeux (1980) and Hosking (1981) can be motivated by the observation that some time series appear to have unbounded spectral densities at the frequency zero, but the spectral densities of first differences appear to vanish at the zero frequency.

In this paper, following Hsu (1997) and Liu (2000), a long memory process will be generated through a Semi-Markov process. The long memory process will display the same autocorrelation behavior like a fractionally integrated autoregressive moving average (ARFIMA) series as the lags between the observations go to infinity. ARFIMA processes can be regarded as a halfway between integrated of order zero, $I(0)$ and integrated of order one, $I(1)$ paradigms. The rest of this section will elaborate on the properties of ARFIMA processes in time and frequency domain.

2.1. Time Domain Approach

The fractionally integrated long-memory process proposed by Granger and Joyeux (1980) and Hosking (1981) can be described as:

$$(1-L)^d \Phi(L) y_t = \Theta(L)\varepsilon_t \quad ;\varepsilon_t \sim i.i.d(0,\sigma^2) \tag{1}$$

where $L^k x_t = x_{t-k}$ for integer k, $\Phi(L)$ and $\Theta(L)$ are polynomials in L with no common roots.

Such a representation has the potential of capturing both the long-term persistence and short-term dynamics at the same time. If $d < 1/2$ and the roots of the polynomials $\Phi(L)$ and $\Theta(L)$ are outside the unit circle, then the time series is stationary and has the usual one-sided MA representation. If $d \geq 1/2$, then the series is nonstationary and when $d = 1$, we have the usual unit-root process.

A process $\{y_t\}$ is called a simple fractionally integrated time-series if we have

$$(1-L)^d y_t = \varepsilon_t, \quad \varepsilon_t \sim i.i.d(0,\sigma^2) \tag{2}$$

When $d < 1/2$ series is stationary and y_t has the Moving average (MA) expression

$$y_t = (1-L)^{-d} \varepsilon_t \tag{3}$$

The autocorrelation function of $\{y_t\}$ is

$$\rho_k = \frac{\Gamma(1-d)\Gamma(d+k)}{\Gamma(d)\Gamma(1-d+k)} \cong \frac{\Gamma(1-d)}{\Gamma(d)} k^{2d-1} \tag{4}$$

as k gets large, where Γ represents the usual gamma function, and the approximation follows from Sheppard's formula, that for large k, $\Gamma(k+a)/\Gamma(k+b)$ is well approximated by k^{a-b}.

$$\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt, \ \alpha > 0 \tag{5}$$

2.2. Frequency Domain Analysis

It has become standard practice for time series analysts to consider differencing their time series to achieve stationarity. By this, they mean one differences to achieve a form of series that can be identified as an Autoregressive Moving Average (ARMA) model. If a series does need differencing to achieve this, it means the undifferenced series has infinite variance. When a series is differenced, one removes the low frequency components. Suppose y_t has spectrum $f_Y(w)$ and $(1-L)^d x_t = y_t$. The lowercase d is the differencing parameter. Then

$$f_Y(w) = |1 - e^{-iw}|^{2d} f_X(w) \tag{6}$$

The power spectrum of a fractional white noise $(1-L)^d x_t = \varepsilon_t$; $\varepsilon_t \sim i.i.d.(0, \sigma^2)$, will be

$$f_X(w) = |1 - e^{-iw}|^{-2d} \frac{\sigma_\varepsilon^2}{2\pi} = [2\sin(w/2)]^{-2d} \frac{\sigma_\varepsilon^2}{2\pi} \cong cw^{-2d}, \ c \in R \text{ as } w \to 0 \tag{7}$$

Therefore the spectral density goes to infinity for $d > 0$ at frequency zero.

3. ARCH Processes and Regime Switching

3.1. Standard ARCH Models

The simplest example from the class of conditionally heteroskedastic models proposed by Engle (1982) is

$$\varepsilon_t = v_t \sqrt{\alpha_0 + \alpha_1 \varepsilon_{t-1}^2}, \qquad v_t \sim iid\ N(0,1) \tag{8}$$

$$Cov(v_t, \varepsilon_s) = 0 \quad \forall s,t, \ \alpha_0 > 0, \ 0 < |\alpha_1| < 1$$

The variance of ε_t conditioned on the past history of $\varepsilon_{t-1}, \varepsilon_{t-2}, \varepsilon_{t-3}, \ldots$ is defined as

$$E(\varepsilon_t^2 \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \tag{9}$$

The conditional variance follows a first order autoregressive process and is denoted by ARCH(1). Moreover, the squared series ε_t^2 follow an AR(1) process with the autocorrelation structure given

$$\rho(k) = \alpha_1^k \; ; \; k = 1,2,3,... \tag{10}$$

In other words an ARCH process is white noise but not independent and identically distributed as seen by the nonlinear relationship of the error terms via second moments.

3.2. Regime Switching

Lastrapes (1989) considers possible shifts in the constant term α_0 given the nature of the world economy. Shifts cause changes in the unconditional variance and thus ARCH process will be nonstationary. Different regimes characterize the different shifts in the operating procedure of the Federal Reserve (FED). The estimation results in reduced values for α_1. Lastrapes' estimations support Diebold's view (1986) that innovations to volatility may exhibit more persistence than actually exists when nonstationarities in the variance process are not taken into account. Allowing for regime shifts in estimating conditional variance can have quantitatively significant effects on persistence of conditional volatility. Therefore, Lastrapes argues that the possibility of changes in unconditional variance should be considered when specifying ARCH models.

Hamilton (1989) documented the apparent success of the switching-regime model to explain the mean growth rate of nonstationary time series. For each nonstationary series, Hamilton assumes that at any point in time the economy might be either in a fast or slow growth phase, with the switch between the two governed by the outcome of a Markov process.

Hamilton and Susmel (henceforth HS) (1994) introduce a parsimonious representation for Markov Regime Switching ARCH (henceforth MRSARCH). Without incorporating the structural shifts, HS find extremely persistent movements in stock price volatility. However, such persistence turns out to be hard to reconcile with the poor forecasting performance of the ARCH process. Following Perron's observation (1989) that changes in regime may give spurious impression of unit roots in characterization of the level of a series, HS explore a specification in which the parameters of an ARCH process occasionally change. Their specification treats the parameter changes as a function of the state as in Cai (1994). HS show that most of the persistence in stock price volatility can be attributed to the persistence of low, moderate and high volatility regimes, which typically last for several years.

3.3. Regime Switching and Long Memory

Long memory processes were mentioned as a possible candidate for persistence in volatility. One might wonder at this point whether there exists a possible relationship between regime switching models and long memory phenomena. Hsu (1997) showed that a Semi-Markov process with a heavy tailed distribution for the waiting times acts like a long memory in the covariance structure as the lags between observations go to infinity. The positive support condition for the original series in our simulations is crucial in the sense that the squares of the error terms are positive, and thus positive support is needed for the values observed between the waiting times. Liu (2000) states the conditions under which regime switching can exhibit long memory. Liu argues that the heavy tail interarrival distribution is the only distribution that gives rise to long memory (pp 143). A fair question may be asked as regards the triggering mechanism of regime switches. According to Liu (2000), a regime may very well be related to certain latent state variables. In the context of financial economics, the regime switching may be due to monetary policy, in the case of interest rates; or correspond to market uncertainty levels as laid out by various major market news, in the case of stock market volatility. When an economic variable hits its threshold, it may trigger a jump in regimes. Liu (2000) models stock market volatility through regime switching and argues for a better fit to the dynamics of the variance process. Hsu (1997) provides a better exposition of the analytics of the regime switching process.

Hsu sets out with a couple of definitions and theorems to prove her case. (Hsu, pp. 11-13).

I follow Hsu's notation in the definitions below:

Definition 3.3.1: Let F and G be cumulative distribution functions (cdfs) with positive support. A sequence $\{\tau_n\}$ is said to be a renewal sequence if $\tau_n = \tau_{n-1} + T_n$ and $\tau_0 = 0$, where $\{T_i : i \geq 2\}$ is an *i.i.d.* sequence of random variables with cumulative distribution function (cdf) F and T_1 has the cdf G and is independent of $\{T_i : i \geq 2\}$.

Definition 3.3.2: A counting process $N(t)$ with respect to a renewal sequence $\{\tau_n\}$ is defined,

$$N(t) \equiv \sum_{n=1}^{\infty} 1_{[0,t]}(\tau_n) = \max\{n \in Z^+ : \tau_n \leq t\} \tag{11}$$

which is the number of transitions in [0,t].

Definition 3.3.3: Let $\{w_n : n \in Z^+\}$ be a Markov process and $\{T_n : n \in Z^+\}$ be a sequence of independent, nonnegative random variables such that the distribution of T_n depends only on w_{n-1}. Let $\{S_n\}$ be the partial sums of the process $\{T_n\}$; that is

$$S_n = \sum_{i=1}^{n} T_i \tag{12}$$

Define $\alpha_t = w_n$ for $S_n \le t \le S_{n+1}$. Then the process $\{\alpha_t : t \in R^+\}$ is called a Semi-Markov process with states $\{w_n\}$ and sojourn times $\{T_n\}$.

Hsu (1997) considers a process $\{\alpha_t\}$ defined by $\alpha_t = w_{N(t)}$, where $\{w_n\}$ is Markovian and $N(t)$ is a counting process. Then the process $\{\alpha_t\}$ is a special case of a Semi-Markov process. Hsu assumes that $\{w_n\}$ is strictly stationary, the mean of the distribution F exists and

$$G(t) = \mu^{-1} \int_0^t (1 - F(x))dx \tag{13}$$

Definition 3.3.4: The forward recurrence time B_t at time t, which is the waiting time from time t to the next transition, is given by:

$$B_t = \tau_{N(t)+1} - t \tag{14}$$

Resnick (1994) provides a proof for the argument that $B_t \sim G(t)$ $\forall t \ge 0$.

Suppose F is a Pareto distribution with the density function

$$f(x) = \frac{\beta x_0^\beta}{x^{\beta+1}} 1_{\{x \ge x_0\}};\ x_0 > 0,\ \beta \in (1,2) \tag{15}$$

The value of β characterizes the tail behavior of the sojourn distribution. The mean of the Pareto distribution is given as $\frac{\beta x_0}{\beta - 1}$, and the variance as $\frac{\beta x_0^2}{(\beta-1)^2(\beta-2)}$. Therefore the mean does not exist for $\beta \le 1$. It is heavy tailed for $\beta < 2$. Then, one can choose $\beta \in (1,2)$ for modeling long memory processes. Hsu (1997) obtained the distribution of the forward recurrence time G by computing

$$1 - F(t) = \int_t^\infty f(x)dx = \begin{cases} (x_0/t)^\beta; & t \ge x_0 \\ 1 & ;t < x_0 \end{cases} \tag{16}$$

$$G(t) = \mu^{-1}\int_0^t \{1 - F(x)\}dx = \begin{Bmatrix} 1 - \beta^{-1}(x_0/t)^{\beta-1}; t \geq x_0 \\ (\beta x_0)^{-1}(\beta - 1)t \quad ; t < x_0 \end{Bmatrix} \tag{17}$$

Then it is easy for us to derive the covariance structure of the Semi-Markov process $\{\alpha_t\}$:

$$\begin{aligned} Cov(\alpha_t, \alpha_{t+h}) &= E(\alpha_t \alpha_{t+h}) - E(\alpha_t)E(\alpha_{t+h}) \\ &= E[E[w_{N(t)} w_{N(t+h)} \mid N(t), N(t+h)]] - \mu^2 \end{aligned} \tag{18}$$

Since w_n's are i.i.d. for $n \in Z^+$,

$$Cov(\alpha_t, \alpha_{t+h}) = (\mu_2 - \mu^2)E(1_{\{N(t)=N(t+h)\}}) \tag{19}$$

where $1_{\{N(t)=N(t+h)\}} = 1$ *if* $N(t) = N(t+h)$, and 0 otherwise, and μ_2 stands for the second moment of the distribution of w_n. Then,

$$\begin{aligned} Cov(\alpha_t, \alpha_{t+h}) &= (\mu_2 - \mu^2)P[N(t) = N(t+h)] \\ &= (\mu_2 - \mu^2)P[B_t > h] = (\mu_2 - \mu^2)(1 - G(h)) \\ &= \begin{Bmatrix} (\mu_2 - \mu^2)\beta^{-1}(x_0/h)^{\beta-1} & ; h \geq x_0 \\ (\mu_2 - \mu^2)\left(1 - (\beta x_0)^{-1}(\beta - 1)h\right) & ; h < x_0 \end{Bmatrix} \end{aligned} \tag{20}$$

The limiting behavior of the covariance function of the Semi-Markov process will be

$$\lim_{h\to\infty} \frac{Cov(\alpha_t, \alpha_{t+h})}{h^{1-\beta}} = \lim_{h\to\infty} \frac{\gamma_\alpha(h)}{h^{1-\beta}} = c, \ t \geq x_0, \ c \in R \tag{21}$$

A simple fractionally integrated long memory process Γ has the limiting covariance structure:

$$\lim_{h\to\infty} \frac{\gamma_\Gamma(h)}{h^{2d-1}} = c, \ c \in R \tag{22}$$

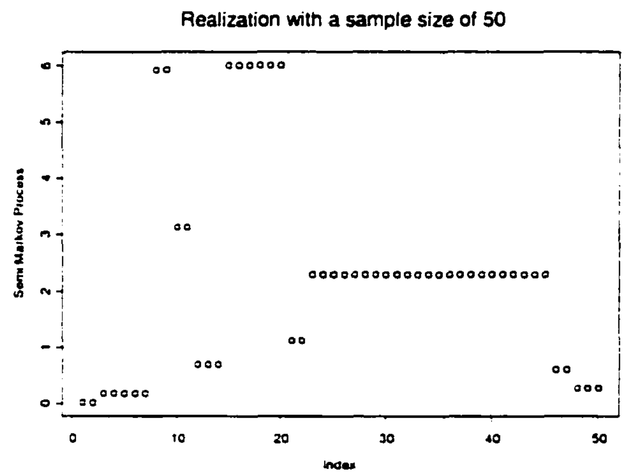
Then all one needs to do in order to create a long memory process with positive support is to generate i.i.d. random variables α_t's from a random number generator with a positive support, i.e. lognormal distribution and embed it into a renewal process where the interarrival times for the process are characterized by a heavy tailed distribution again with positive support, i.e. Pareto distribution. The relationship between d and β can be derived simply as:

$$2d - 1 = 1 - \beta \Rightarrow \beta = 2(1 - d) \tag{23}$$

Then, one can calibrate the autocorrelation function of the long memory process created by regime switching to another fractionally integrated long memory process. The relationship in equation (23) accords with Lemma 1.1. in Liu (2000). Our illustration here only serves as a concrete example of Liu's arguments.

A Possible realization for the Semi-Markovian process for fifty observations is in Fig. 1. Each realization is from a lognormal$(\mu = 0, \sigma^2 = 5)$ distribution and the waiting times are Pareto$(x_0 = 0.99, \beta = 1.2)$ and the differencing parameter d of the ARFIMA process the simulations were calibrated to is 0.4.

FIG.1 SEMI-MARKOV PROCESS $(\log normal(0,5), Pareto(0.99,1.2))$



4. Data Generation and Calibration

The simulations of standard ARCH, Markov Regime Switching ARCH (MRSARCH) and Long Memory processes are calibrated to have parameter values to equalize not only unconditional variances of the three processes but also the sum of the first five autocorrelations of the squared series. In order to be able to compare the sole effect of the regime switching on long memory tests, the parameters for the three processes were chosen to have the same behavior in levels and autocorrelation structures. That was crucial to extract the impact of regime switching for ARCH and MRSARCH processes. The sum of the first five autocorrelations rather than the first order autocorrelations was preferred on ad hoc basis to account for the differing decay factors for long memory and short memory processes.

4.1. Standard ARCH

Let $\varepsilon_t = u_t\sqrt{h_t}$, $u_t \sim i.i.d.\ N(0,1)$ and $E(\varepsilon_t^2 \mid \varepsilon_{t-1}, \varepsilon_{t-2}, \ldots) = \alpha_0 + \alpha_1\varepsilon_{t-1}^2$ where $|\alpha_1| < 1$ and $\alpha_0 > 0$, and N is the normal distribution with the necessary parameters. In the simulations α_1 will be chosen in accordance with the selection of the parameter d, and α_0 will be chosen to calibrate unconditional variances. The maximum first order autocorrelation that can be obtained for the long memory process through regime switching is 0.33. ARCH process with first order, ARCH (1), will be generated recursively, assuming $\varepsilon_0 = 0$. MRSARCH and ARCH(1) processes will be generated with sample sizes of 500, 750 and 1000 with 1000 repetitions. The initial 200 values for each repetition in the Monte Carlo simulations will be discarded to avoid a possible adverse effect of the initial value.

4.2. Markov Regime Switching ARCH Model

The Markov Chain in the simulations has a finite number of states, and to make it simple I choose 2 states. The states 1 and 2, and the stationary transition probabilities for the Markov Chain are characterized by the transition matrix

$$P = \begin{bmatrix} p & 1-p \\ 1-q & q \end{bmatrix} \qquad (24)$$

where p is the transition probability from state 1 unto itself, and similarly, q is the transition probability from state 2 unto itself, $(1-q)$, and $(1-p)$ are transition probabilities between states.

The transition matrix P has all its rows summing to one as expected from a Markov Chain. The vector of ergodic probabilities, the unconditional probability of the system is at steady state, is

$$\begin{bmatrix} (1-q)/(2-p-q) \\ (1-p)/(2-p-q) \end{bmatrix} \qquad (25)$$

where the first row refers to the ergodic probability for the first state and the last row to the second state. The ergodic probabilities will be used to determine from which state the regime switching process starts after one fixes the values for p and q. I generate a uniform random variable $Y \sim U[0.1]$, and if Y is greater than $\dfrac{1-q}{2-p-q}$, then the system starts at state 2, and at state 1 if the reverse holds. I choose the p, q pairs as $(0.3,0.3), (0.5,0.5), (0.7,0.7), (0.9,0.9)$ to examine how the state transition probabilities affect the persistence in a MRSARCH process. I try to keep the p, q parameters the same, as other parameterizations have not proved to be very fruitful for interpretation. The ARCH(1) processes for each state are given as:

$$\varepsilon_t^{(i)} = u_t \sqrt{h_t^{(i)}}\ ;\ u_t \sim iid\ N(0,1),\ i \in \{1,2\}. \tag{26}$$

$$\varepsilon_t^{(1)2} = \alpha_0^{(1)} + \alpha_1^{(1)} \varepsilon_{t-1}^{(1)2},\ \varepsilon_t^{(2)2} = \alpha_0^{(2)} + \alpha_1^{(2)} \varepsilon_{t-1}^{(2)2}$$

The superscripts in parentheses represent the state, $\alpha_0 > 0,\ \alpha_1 \in (0.1)$, and $\left(\alpha_0^{(2)}, \alpha_1^{(2)}\right) = m\left(\alpha_0^{(1)}, \alpha_1^{(1)}\right),\ m \in R^+$. The ARCH (1) process for the second state has parameter structure just a constant multiple of the first state. In order to represent the second state as the high variance state, m is 2, and α_0 and α_1 are chosen to equate the unconditional variance and the sum of first five autocorrelations. In the simulations the error term at time zero will be assumed to be zero and the initial 200 observations in the simulation will be discarded. In order to highlight the issue of regime switching ARCH, it is worth looking at how the process is simulated on a small example:

Suppose we are going to simulate n observations through MRSARCH. Let the state vector for n observations be: $[1\ 1\ 2\ 2\ 2\ 2..1\ 1]_{(1\times n)}$. Then the first three simulated values are going to be:

$$\varepsilon_1 = u_1 \sqrt{\alpha_0^{(1)} + \alpha_1^{(1)} \varepsilon_0^2},\ \varepsilon_2 = u_2 \sqrt{\alpha_0^{(1)} + \alpha_1^{(1)} \varepsilon_1^2},\ \varepsilon_3 = u_3 \sqrt{\alpha_0^{(2)} + \alpha_1^{(2)} \varepsilon_2^2} \tag{27}$$

This algorithm will allow for a switch of the volatility process for the whole series.

4.3. Simulating a Semi-Markov Process and Long Memory

We need to construct a long memory process with a positive support to make the comparison possible with ARCH and MRSARCH. Long memory processes in general have support on the real line. However, variance is always positive and therefore the series needs to have positive values. I use a renewal process with a lognormal distribution to generate a positive valued long memory series. Pareto distribution for the waiting times can be simulated easily from a uniform random number generator by using the inverse cdf method:

$$\text{Generate } Z \sim U[0,1],\ \text{then } P = \frac{x_0}{(1-Z)^{1/\beta}} \sim Pareto(x_0, \beta) \tag{28}$$

At each renewal I generate a lognormal random variable with parameters $(\mu, \sigma^2) = (0.5)$ and the series has that value until the new renewal. The density function for a lognormal random variable is:

$$f(x \mid \mu, \sigma^2) = \frac{1}{\sqrt{2\pi}\sigma} \frac{e^{-(\log x - \mu)^2 / 2\sigma^2}}{x},\ 0 \le x < \infty,\ -\infty < \mu < \infty,\ \sigma > 0 \tag{29}$$

The waiting time the process stays at any lognormal variable is denoted by $\lfloor P \rfloor$. P is a random variable with a Pareto distribution with parameters $(x_0, \beta) = (1, 2(1-d))$. d is the fractional differencing parameter for the ARFIMA process. The value for β is chosen to equate the asymptotic autocorrelation functions for a regime switching process and an ARFIMA process with differencing parameter, d. The function $\lfloor ... \rfloor$ stands for the smallest integer value greater than or equal to P. The process generated will be:

$$\varepsilon_t = u_t \sqrt{v_{N(t)}}\ ,\ u_t \sim iid\ N(0,1)\ ;\ Cov(u_t, v_{N(t)}) = 0 \tag{30}$$

$v_t \sim \log$ normal (μ, σ^2), and $N(t)$ is a counting process with Pareto(x_0, β) interarrival times. The autocorrelation function of $v_{N(t)}$ process will carry the features of an ARFIMA process with differencing parameter d. The differencing parameter values that will be examined are $0.1, 0.3, 0.4$ and 0.49. A higher d value will result in more persistence in variance.

4.4. Calibration

This section will calibrate the process parameters so that each of the three processes has the same unconditional variance and the same sum of the first five autocorrelations for the squared series. All the derivations for the corresponding variances and autocorrelations are derived in the Appendix.

I set the differencing parameter for the ARFIMA process as $d \in \{0.1, 0.3, 0.4, 0.49\}$ which translates into $\beta \in \{1.8, 1.4, 1.2, 1.02\}$. The value for α_1 was calculated to be the element of the set $\{0.323, 0.457, 0.54, 0.64\}$, which translates into $\alpha_1^{(1)} \in \{0.17, 0.25, 0.261, 0.2865\}$. With four different values for d, four (p,q) pairs and three different sample sizes of $500, 750$ and 1000, the paper will analyze 48 different cases to display possible effects of each parameter and the interplay between them in Monte Carlo simulations.

The values for α_0 and $\alpha_0^{(i)}$ for $i \in \{1,2\}$ are adjusted accordingly to equate the variance and the autocorrelations for each of the three processes. In all simulations of long memory through the Semi-Markov process, I keep the x_0 parameter for the Pareto distribution equal to 0.99 to maximize the first order autocorrelation of the long memory process. One other aspect of the simulations is that for each parameterization I use the same iid noise sequence.

5. Testing through MRR and GPH

We first illustrate the testing procedures used to identify long memory series. The first part is the Modified Rescaled Range (MRR) statistic distinguishing long memory from short memory, and the second part makes use of the Geweke and Porter-Hudak (GPH) test for long memory in the frequency domain to identify the significance of the differencing parameter, d.

5.1. A Robust Nonparametric Test for Long Memory

To detect long-range or strong dependence and also to take into account possible short memory we use Lo (1991)'s " Modified Rescaled Range (MRR)" test statistic. This corrects for short-term memory without taking too strong a position on what form it takes. Its limiting distribution is invariant to many forms of short-range dependence, and yet it is still sensitive to the presence of long-range dependence. The modified R/S statistic is defined as

$$Q_n = \frac{1}{\hat{\sigma}_n(q)}\left[\max_{1\le k\le n}\sum_{j=1}^{k}(x_j-\bar{x}_n)-\min_{1\le k\le n}\sum_{j=1}^{k}(x_j-\bar{x}_n)\right] \tag{31}$$

where

$$\hat{\sigma}^2_n = \hat{\sigma}^2_X + 2\sum_{j=1}^{q} w_j(q)\hat{\gamma}_j;\quad w_j(q) = 1-\frac{j}{q+1} \tag{32}$$

$$\hat{\sigma}^2_X = \frac{1}{n}\sum_{j=1}^{n}(x_j-\bar{x}_n)^2$$

and $\hat{\gamma}_j$ is the sample autocovariance estimator. According to the data dependent formula by Andrews (1991), the truncation lag must be chosen following

$$q = [k_n];\ k_n \equiv \left(\frac{3n}{2}\right)^{1/3}\left(\frac{2\hat{\rho}}{1-\hat{\rho}^2}\right)^{2/3} \tag{33}$$

The [.] operator denotes the greatest integer less than or equal to k_n, and $\hat{\rho}$ is the estimator for the first order autocorrelation. There is no clear choice of q in small samples. Andrews (1991) proves in a theorem (p. 1291) that $\frac{1}{\sqrt{n}}Q_n \rightarrow_d V$.

The distribution function is given explicitly by Kennedy (1976) and Siddiqi (1976)

$$F_V(v) = 1 + 2\sum_{k=1}^{\infty}(1-4k^2v^2)\exp(-2k^2v^2) \tag{34}$$

The null hypothesis for MRR test statistic is that the series has short memory. When the test statistic exceeds the chosen critical value, then it is concluded that the series exhibits long memory.

5.2. Estimation for Long Memory in Frequency Domain

Geweke and Porter-Hudak (1983) suggested a semi-parametric estimator of the fractional differencing parameter d that is based on a regression of the ordinates of the log spectral density on low frequencies. The estimator exploits the theory of linear filters to write the process:

$$(1-B)^d x_t = u_t \,;\; u_t \sim i.i.d.(0, \sigma^2) \tag{35}$$

where x_t is a stationary linear process with a spectral density function $f_U(w)$, which is finite and continuous on the interval $[-\pi, \pi]$. The spectral density of x_t is:

$$f_X(w) = \left(\frac{\sigma^2}{2\pi}\right)\left[4\sin^2(w/2)\right]^{-d} f_U(w) \tag{36}$$

Then, by taking the logarithms, adding and subtracting $\ln(f_U(0))$ on both sides

$$\ln(f_X(w)) = \ln\left[\frac{\sigma^2 f_U(0)}{2\Pi}\right] - d\ln\left[4\sin^2(w/2)\right] + \ln\left[\frac{f_U(w)}{f_U(0)}\right]$$

Suppose that a sample $\{x_t\}$ of size T is available. Let $w_{j,T} = \frac{2\pi j}{T}$; $j = 0,1,2,\ldots,T-1$ denote the harmonic ordinates and $I(w_{j,T})$ denote the periodogram at those ordinates. Evaluating the latter equation at $w_{j,T}$ and rearranging

$$\ln\{I(w_{j,T})\} = \ln\left[\frac{\sigma^2 f_U(0)}{2\pi}\right] - d\ln\left[4\sin^2(w_{j,T}/2)\right] + \ln\left[\frac{f_U(w_{j,T})}{f_U(0)}\right] + \ln\left[\frac{I(w_{j,T})}{f_X(w_{j,T})}\right] \tag{37}$$

The intercept is $\ln\left[\frac{\sigma^2 f_U(0)}{2\pi}\right]$ plus the mean of $\ln\left[\frac{I(w_{j,T})}{f_X(w_{j,T})}\right]$. The latter term is considered to be the error in the regression. The one before the last term in (37) is negligible as attention is confined to frequencies near zero. The proposed estimator is the slope coefficient in the least squares regression of $\ln\{I(w_{j,T})\}$ on a constant and $\ln[4\sin^2(w_{j,T}/2)]$; $j = 1,2,\ldots,g(T)$, where $g(T)$ is chosen to be $T^{0.5}$ following GPH (1983). Then one can employ the usual t test for the significance of the differencing parameter.

6. Simulation Results

We examine the finite sample properties of the GPH and MRR tests, by analyzing the performance of those tests with different data generating mechanisms via Monte Carlo methods. Critical values used in the Monte Carlo simulations are based on the asymptotic distribution of the tests in consideration.

The outcomes of all simulations are illustrated in figures. The visual display of results allow for a better empirical assessment of the evolution of ARCH, MRSARCH and long memory process in time (Fig.2) and the behavior of their sample autocorrelation functions (Fig.3).

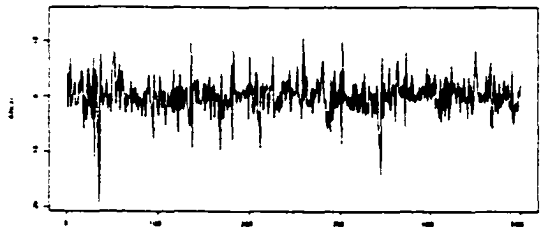
The simulation results (Fig.4) suggest that the MRR test is conservative in the sense that it tends to reject the null of short memory less frequently than the nominal 5% significance level. According to the calibration procedure the autoregressive parameter of the ARCH process increases as the differencing parameter increases. Intuitively we may expect that the higher the autoregressive parameter, or the higher d is, the lower the size of MRR test. However, the MRR test is robust to the autoregressive parameter in the conditional variance, because the truncation lag parameter that controls the amount of autocorrelation to be discounted is being adjusted according to the dependence in the data. The GPH test for ARCH processes (Fig.4) has size close to its nominal significance level of 5%. The behavior of GPH varies little across different ARCH processes with different autoregressive parameters. Overall, GPH test has lower size than MRR. The extent of frequencies to be included in regression in frequency domain is fixed at the square root of the sample size and that impedes its ability to adjust for serial correlation. When it comes to the simulated long memory processes through a Semi-Markov process, the power of the MRR test (Fig.5) increases with the differencing parameter. The same pattern arises for the GPH test (Fig.5). The power of the tests for the same long memory differs, however, granting more rejection of the null of short memory for various differencing parameter values and sample sizes.

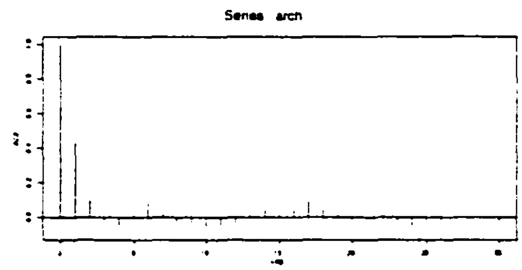
FIG.2

TIME PATH OF ARCH, MRSARCH, AND LONG MEMORY PROCESSES

FIG.3

AUTOCORRELATION FUNCTIONS OF SQUARED ARCH, MRSARCH AND LONG MEMORY PROCESSES

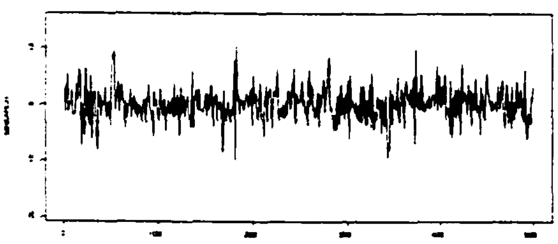


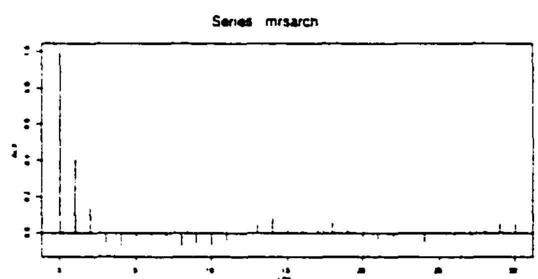


ARCH $(\alpha_0 = 4.38, \alpha_1 = 0.64, n = 500)$

MRSARCH

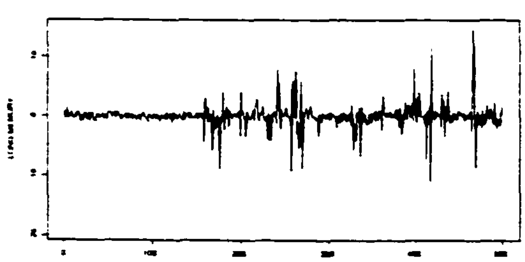
$(\alpha_0^{(1)} = 4.0038, \alpha_1^{(1)} = 0.2865, p = q = 0.9, n = 500)$





Simulated Long Memory

$(x_0 = 0.99, \beta = 1.02, \log normal(0.5), n = 500)$



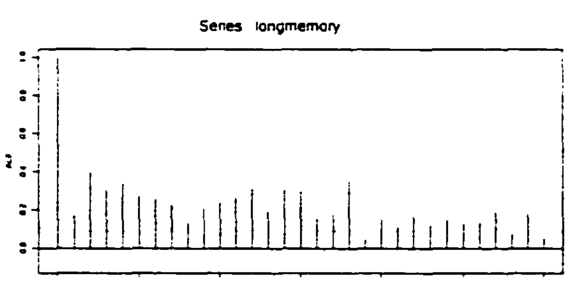


FIG.4 ARCH AND LONG MEMORY TESTS

All processes have the same unconditional variance and equal sum for the first 5 autocorrelations for a given differencing parameter. The differencing parameter stands for the long memory process to which the sum of the first five autocorrelations is calibrated. Each simulation is done on 1000 repetitions. Nominal size is 95%.

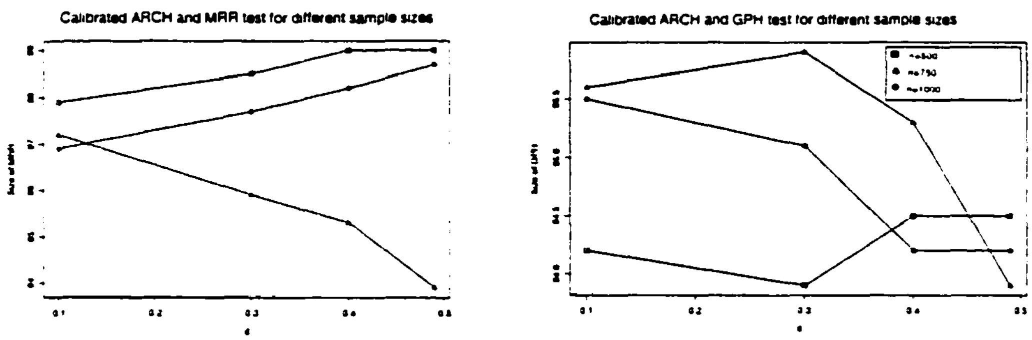


FIG.5 LONG MEMORY AND LONG MEMORY TESTS

The parameters for Pareto and LogNormal distributions are as given in the Table. A common value of $x_0 = 0.99$ was used for each simulation. Nominal size is 95%.

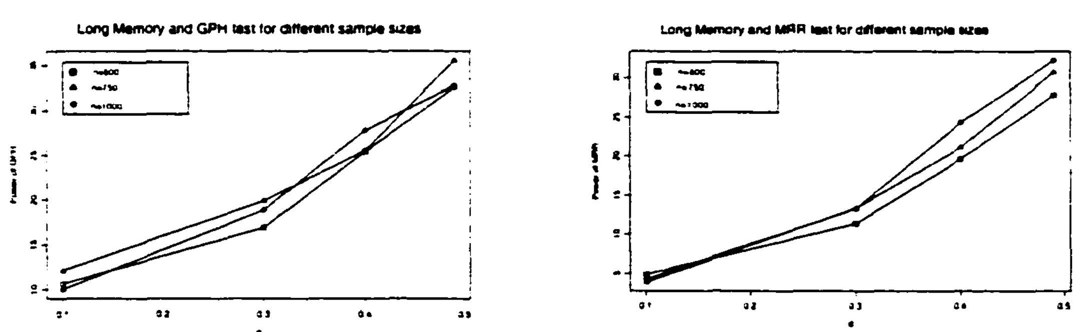
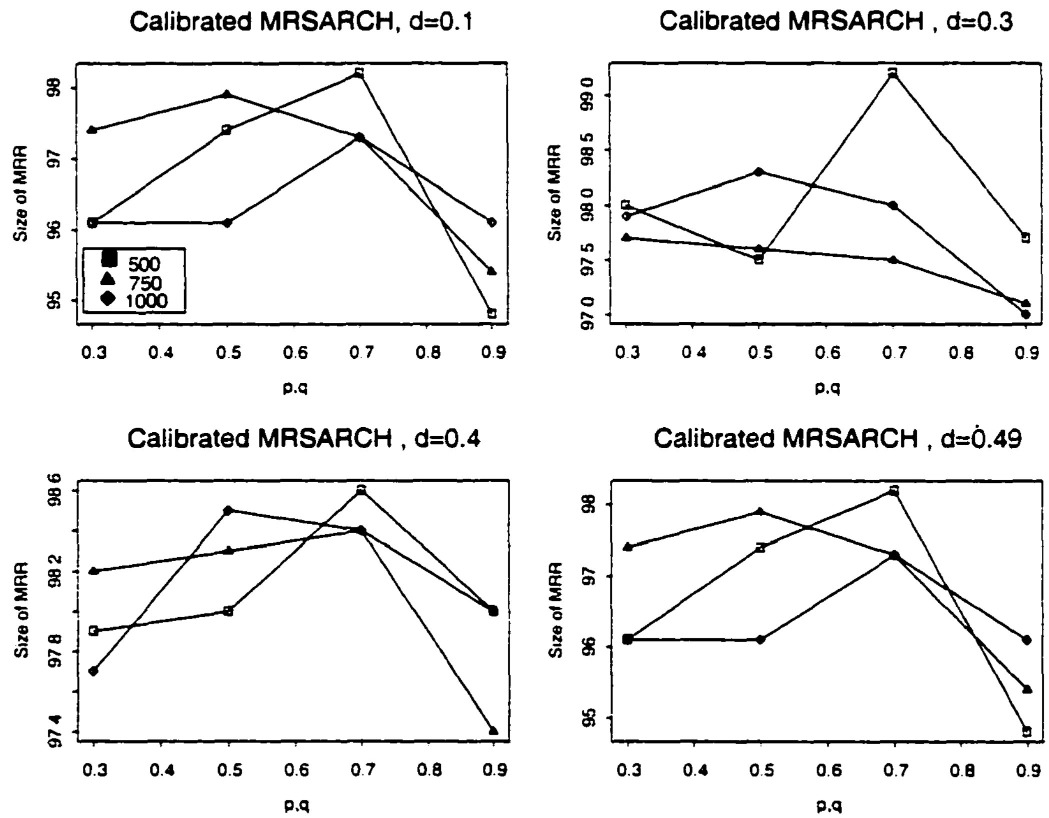


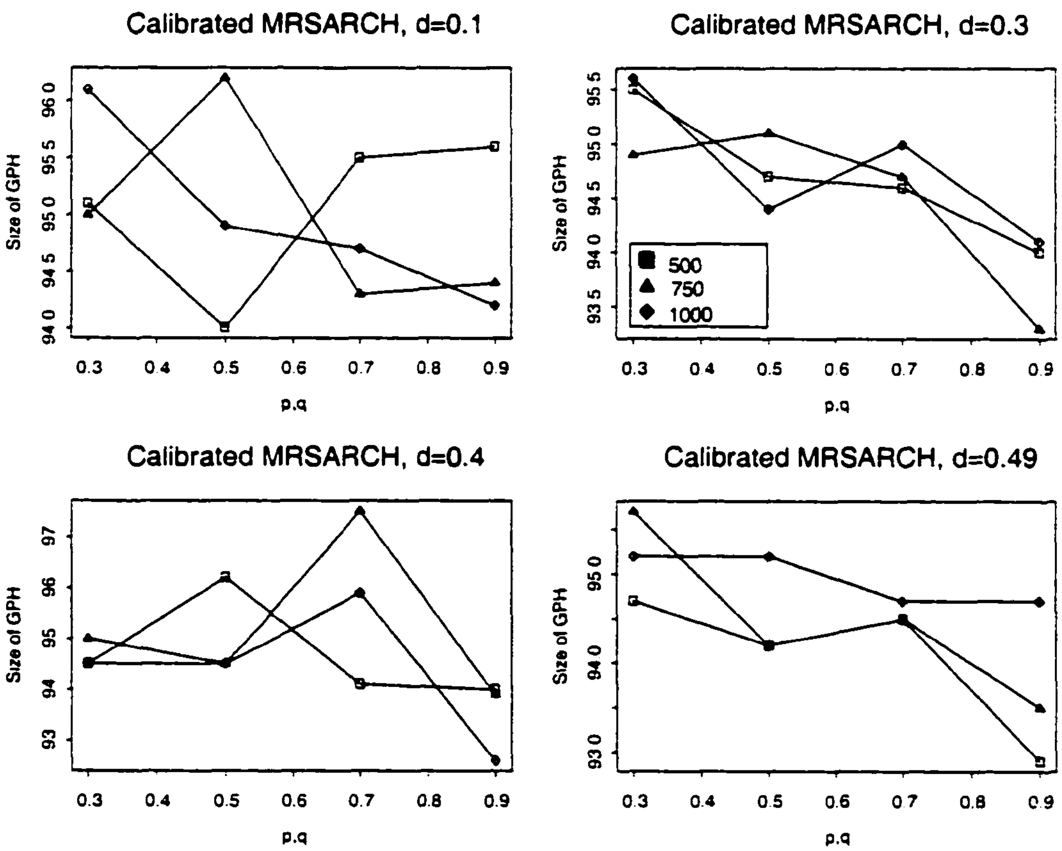
FIG.6 MRSARCH and MRR TEST

Across a given differencing parameter all MRSARCH processes have the same unconditional variance and equal sum for the first five autocorrelations. The parameter $\alpha_1^{(1)}$ is calibrated given the sum of autocorrelations for the long memory process and $\alpha_0^{(1)}$ is calibrated after the choice of $\alpha_1^{(1)}$, given (p,q) pair. Nominal size is 95%.



The same over-acceptance behavior is observed for MRR test in MRSARCH processes. (Fig.6). Almost all sizes of the test are higher than a 95% nominal size level. For various differencing parameter and sample sizes, MRR test has the highest rejection rate with the (p,q) pair of (0.9,0.9) 9 out of 12 cases, which corresponds to the highest persistence considered in Markov Regime Switching. It is not possible to see an inverse relationship between the state transition probabilities and the size of MRR test from 0.3 through 0.9. The behavior of MRR seems unpredictable for small and moderate p,q values, but the dramatic change for 0.9 is pretty obvious. In other words, as the switches across states occur less frequently, then the process tends to develop more persistence in variance leading to a higher rejection rate for the MRR test. But one observation is clear: As persistence in MRSARCH process increases so does the rejection rate due to false presumption that it has long memory features. As for the GPH tests (Fig.7) for various differencing parameter values, the size of GPH follows a nonlinear pattern with changing sample sizes, but the lowest size still occurs at p,q pair of (0.9,0.9) in 10 out of 12 cases.

FIG.7 MRSARCH and GPH TEST



The last two figures (Fig.8 And Fig.9) display the persistence associated with Markov regime switching. MRSARCH almost always has a lower size for the long memory tests, supporting the notion that switching processes attribute more persistence to the underlying process

FIG.8 COMPARISON of ARCH AND MRSARCH in MRR TEST

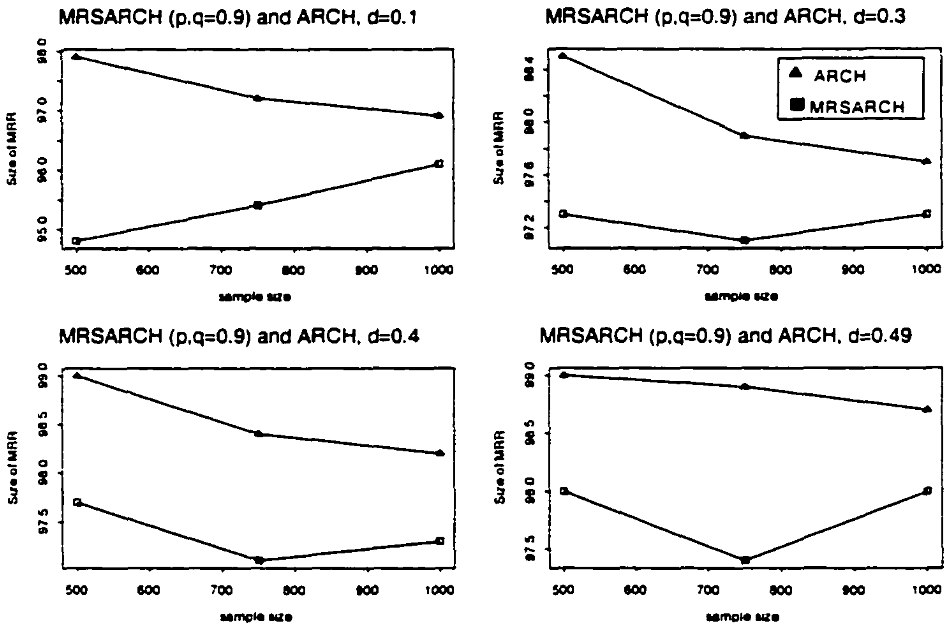
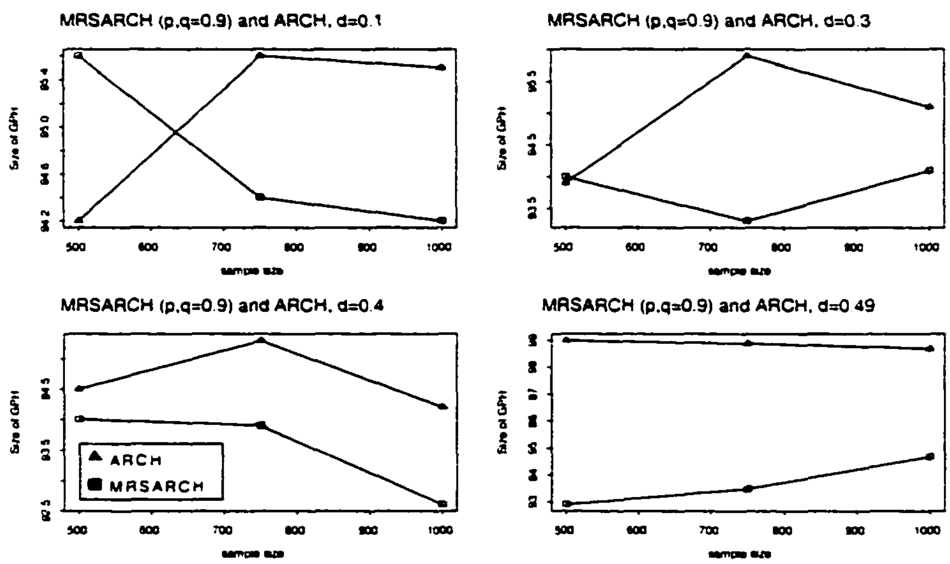


FIG.9 COMPARISON of ARCH AND MRSARCH in GPH TEST



For the figures illustrated, the ARCH process when squared had a mean of 11.84, MRSARCH 11.89, pretty close suggesting correct calibration. As for the long memory process, although the individual means of the squared series varied, the mean of the 500 replications resulted in a mean value of 11.63, again validating ergodicity and our calibration procedure.

Conclusion

The finite sample properties of long memory tests are investigated using Monte Carlo methods. The size and power of MRR and GPH tests are examined under various data generating mechanisms. Three different processes, ARCH, Markov Switching ARCH and Long Memory series are generated using identical noise series. To better assist in the comparison of various tests, all three processes were calibrated to have equal unconditional variance and identical sum for the first five autocorrelations. The simulations in this study provide better information than Cheung (1993) as to the comparison of the size and power of MRR and GPH tests, since other factors, autocorrelations and unconditional variance are accounted for. The performance of long memory tests on a two state Markov Chain is provided. It also adds to Cheung's analysis by providing the performance of long memory tests in the presence of structural changes. The sizes of MRR and GPH tests are not corrected through Bootstrap simulations, as that was a secondary objective for our purposes.

This study has its common feature with Hsu (1997) and Liu (2000) by its expository analysis of how regime switching processes tend to behave like a long memory process. The loss in the size for MRR and GPH tests when the persistence to stay in a given state is high in MRSARCH processes is observed. At the other end of the spectrum, a Semi-Markov process with Pareto waiting times illustrates how persistence could lead to long memory type behavior.

Both MRR and GPH tests are found to be quite robust to ARCH and MRSARCH effects. The simulation outcomes suggest that MRR test is more conservative in the sense that it tends to reject the null more frequently than the nominal 95% significance level. MRR results turn out to be more conservative than other similar studies. (Cheung, 1993) In both tests MRSARCH always had the smallest size confirming the common argument that switching processes create persistence in the variance. The power of the MRR test for regime switching stochastic volatility increases with the differencing parameter. As for GPH test, the change in the differencing parameter significantly affects the power .

Overall, Monte Carlo results suggest the importance of persistence in regime switching behavior for creating long memory features. An important analogy is in place here: The simulation results suggest that as the differencing parameter d is increased, the process acts more like a long memory in the Semi-Markovian process. However, β decreases with increasing d, and that prolongs the expected waiting time in the switching regime. Thus, one is more likely to stay at a given realization. Similarly, in a MRSARCH process, as p,q pair decreases one is more likely to remain at a given state and thus more persistence arises. Our simulations attest empirically to Liu's findings on changing persistence with tail index of the waiting distribution. Liu observes that although the Markov Chain regime switching model does not truly represent stock market volatility, the persistence to stay in a given state is above 0.9. Our findings from the simulation also indicate that the highest persistence is obtained with high transition probability of returning to the same state.

APPENDIX:

In what follows, I derive the unconditional variances and the autocorrelations for the ARCH, MRSARCH and Long Memory processes. An ARCH process is defined as:

$$\varepsilon_t = u_t\sqrt{h_t}\ ,\ u_t \sim i.i.d.\ N(0,1),\ \text{and}\ E(\varepsilon_t^2) = \alpha_0 + \alpha_1 E(\varepsilon_{t-1}^2) \tag{A.1}$$

The unconditional variance of ε_t is equivalent to that of ε_{t-1}, and hence

$$V(\varepsilon_t) = \frac{\alpha_0}{1-\alpha_1} \tag{A.2}$$

The autocorrelation at the i-th lag for the ε_t^2 sequence is $\alpha_1^{(i)}$.

For a MRSARCH process, the variance and the autocorrelation are just a weighted average of the low and high volatility states with weights equal to the ergodic probabilities. That is

$$V(MrsArch) = \frac{1-q}{2-p-q}\frac{\alpha_0^{(1)}}{1-\alpha_1^{(1)}} + \frac{1-p}{2-p-q}\frac{\alpha_0^{(2)}}{1-\alpha_1^{(2)}}. \tag{A.3}$$

Letting $\alpha_0^{(2)} = 2\alpha_0^{(1)}$ and $\alpha_1^{(2)} = 2\alpha_1^{(1)}$

$$V(MrsArch) = \frac{1-q}{2-p-q}\frac{\alpha_0^{(1)}}{1-\alpha_1^{(1)}} + \frac{1-p}{2-p-q}\frac{2\alpha_0^{(1)}}{1-2\alpha_1^{(1)}}. \tag{A.4}$$

Similarly

$$\rho(i) = \frac{1-q}{2-p-q}(\alpha_1^{(1)})^i + \frac{1-p}{2-p-q}(2\alpha_1^{(1)})^i \tag{A.5}$$

For the Semi-Markov process that approximates long memory, I write the generated series as:

$\varepsilon_t = u_t\sqrt{v_{N(t)}}$, $u_t \sim i.i.d.N(0,1)$, and $Cov(u_t, v_{N(t)}) = 0$

$v_s \sim$ lognormal(μ, σ^2) , $s \in Z^+$ and N(t) is a counting process with Pareto(x_0, β) interarrival times. In this case

$$V(\varepsilon_t) = E(u_t^2 v_{N(t)}) = E(u_t^2)E(v_{N(t)}) = E(E(v_{N(t)}) \mid N(t)) = e^{\mu+\sigma^2/2} \quad \text{(A.6)}$$

As for the correlation, I calculate the variance and covariance of ε_t^2 sequence.

$$V(\varepsilon_t^2) = E(u_t^4 v_{N(t)}^2) - (E(\varepsilon_t^2))^2 = 3E(E(v_{N(t)}^2 \mid N(t)) - V(\varepsilon_t)^2 = 3e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2} \quad \text{(A.7)}$$

$$\begin{aligned}
Cov(\varepsilon_t^2, \varepsilon_{t+1}^2) &= E(\varepsilon_t^2 \varepsilon_{t+1}^2) - E(\varepsilon_t^2)E(\varepsilon_{t+1}^2) \\
&= E(u_t^2 v_{N(t)} u_{t+1}^2 v_{N(t+1)}) - E(v_{N(t)})E(v_{N(t+1)}) \\
&= Cov(v_{N(t)}, v_{N(t+1)}) \\
&= E(E(v_{N(t)} v_{N(t+h)} \mid N(t), N(t+h)) - E(E(v_{N(t)} \mid N(t))^2 \\
&= (E(v_s^2) - e^{2\mu+\sigma^2})E(1_{\{N(t)=N(t+1)\}}) \\
&= (\mu_2 - \mu^2)P(B_t > 1) = (e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2})\beta^{-1}x_o^{\beta-1}
\end{aligned} \quad \text{(A.8)}$$

$$\rho(1) = \frac{Cov(\varepsilon_t^2, \varepsilon_{t-1}^2)}{V(\varepsilon_t^2)} = \frac{(e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2})\beta^{-1}x_o^{\beta-1}}{3e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2}} = \frac{e^{\sigma^2} - 1}{3e^{\sigma^2} - 1}\beta^{-1}x_o^{\beta-1} \quad \text{(A.9)}$$

Since $\beta \in (1,2)$ and $x_0 < 1$, $\rho(1)$ is always less than 1, and has an upper bound 1/3.

I equate the sum of the first five autocorrelations of the long memory process to the ARCH process and calculate the ARCH and MRSARCH parameters for different (p, q) pairs.

ARCH	MRSARCH	LONG MEMORY
$\sum_{i=1}^{5} \alpha_1^i$	$\sum_{i=1}^{5}\left[\frac{1-q}{2-p-q}(\alpha_1^{(1)})^i + \frac{1-p}{2-p-q}(2\alpha_1^{(1)})^i\right]$	$\sum_{i=1}^{5}\frac{e^{\sigma^2}-1}{3e^{\sigma^2}-1}\beta^{-1}\left(\frac{x_0}{i}\right)^{\beta-1}$

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CHAPTER 2
STRUCTURAL VARs OF REAL EXCHANGE RATES: WHAT HAVE WE LEARNED? EVIDENCE FROM G-7 COUNTRIES

1. Introduction

This paper takes up the question: do structural vector autoregressions (SVARs) of real exchange rates provide us with reliable information as to the driving forces behind one of the mostly examined variables in international finance? There may be a couple of reasons why one would be interested in the answer to such a question. Firstly, this would help assess the relative merits of vector autoregressions (VARs) with long run restrictions within the framework developed by Blanchard and Quah (1989). Secondly, using variance decompositions it helps us to understand the relative importance of different types of shocks on the real exchange rates. It also may be relevant to empirically test the validity of real exchange rate models through the use of impulse response functions.

SVARs of real exchange rates is one of the quite often-used methods of examining real exchange rate behavior. Applications of SVARs to study real exchange rate dynamics differ according to the variables used in the decomposition, the types of shocks that are being modeled and whether the decomposition is supported by a theoretical model of exchange rate movements. Lastrapes (1992) estimates a bivariate SVAR of real and nominal exchange rates and identifies the transient and permanent components. Evans and Lothian (1993) estimate a trivariate system of real exchange rate, domestic inflation and foreign inflation with the same set of transitory and permanent shocks as in Lastrapes (1992). The seminal paper by Clarida and Gali (1995) is the first to decompose the permanent shocks into supply and demand shocks. Clarida and Gali present an open economy macro model that can be used to interpret the trivariate VAR of relative real gross domestic product (GDP), real exchange rate and relative prices. Enders and Lee (1997) differentiate between the effects of real and nominal shocks using the well-known Dornbusch (1976) 'overshooting' model of a small economy with real and nominal exchange rates in their decomposition approach. Rogers (1999) enriches the VAR with five variables and five shocks. The interesting feature of Rogers is that it partitions the monetary shocks into money demand and money supply shocks. Despite the different specifications among these SVARs, some common perceptions have emerged from this literature regarding how real exchange rates have been determined in the post-Bretton Woods period. First, real shocks dominate nominal shocks

(Lastrapes, 1992; Enders and Lee, 1997;Clarida and Gali, 1995). Second, demand shocks are much more important than the supply shocks in explaining real exchange rate fluctuations (Enders and Lee, 1997; Clarida and Gali, 1995). Third, sticky price models fare well in general and there is evidence for an overshooting effect of the nominal shock in accord with the predictions of such models (Lastrapes, 1992; Enders and Lee, 1997; Clarida and Gali, 1995). Fourth, the role of monetary shocks increases when more shocks and more variables are incorporated for the structural estimation (Rogers, 1999).

In the remainder of this paper I take a closer look at the shock identification process in SVAR modeling, using the same theoretical model as Clarida and Gali (1995) and using data from the G-7 countries, with the U.S. serving as the home country. I find that the estimated responses of the system to the supply, demand and nominal shocks identified by the theoretical restrictions are not as supportive of the theory as the previous literature has suggested. More specifically, the short run and the long run predictions of the theoretical model are not supported by the impulse response functions for the variables used in the trivariate SVAR system.

2. A Dynamic Exchange Rate Model

Clarida and Gali (1995) base their empirical analysis upon a stochastic version of the two-country, rational expectations open macro model developed by Obstfeld (1985). The model exhibits the standard Mundell-Fleming-Dornbusch results in the short-run, with prices adjusting sluggishly to demand, supply, and nominal shocks. But it also embodies the long-run properties that characterize the macroeconomic equilibrium in the open economy once prices fully adjust to all shocks. The model is presented below. All variables are expressed in terms of home relative to foreign levels and all variables except for interest rates are in logarithmic form.

IS Equation: $y_t^d = d_t + \eta(s_t - p_t) - \sigma[i_t - E_t(p_{t+1} - p_t)]; \eta > 0, \sigma > 0$ (1)

LM Equation: $m_t - p_t = y_t - \lambda i_t; \lambda > 0$ (2)

Price Setting Equation: $p_t = (1-\theta)E_{t-1}p_t^e + \theta\, p_t^e; \theta \in [0,1]$ (3)

Interest Parity Equation: $i_t = E_t(s_{t+1} - s_t)$ (4)

and

Specification of the Exogenous Processes:

$$\begin{aligned} \Delta y_t^s &= a(L)\varepsilon_{st} \\ \Delta d_t &= b(L)\varepsilon_{dt} \\ \Delta m_t &= c(L)\varepsilon_{nt} \end{aligned} \qquad (5)$$

where y_t^d is the relative demand for output, y_t^s is the relative supply of output, s_t is the nominal exchange rate, p_t is the relative price level, i_t is the relative nominal interest rate, m_t is the relative nominal money supply, and p_t^e is the market-clearing relative price level.

Equation (1) is an open economy IS equation in which the demand for home output relative to foreign output is increasing in the real exchange rate q_t, defined as $s_t - p_t$, and relative demand d_t, and is decreasing in the real interest differential in favor of the home country. Equation (2) is the standard LM equation. Equation (3) is the price setting equation, with the price level in period t being an average of the market clearing price that is expected in $t-1$, and the price that would actually clear the market in period t. The parameter θ measures the degree of price flexibility. When $\theta = 1$, prices are fully flexible. Equation (4) is the interest parity condition.

The stochastic processes driving the exogenous variables, y_t^s, d_t and m_t, are given in (5). $\varepsilon_s, \varepsilon_d$, and ε_n are supply, demand, and nominal shocks, respectively, which are serially and mutually uncorrelated, zero-mean and finite-variance processes. I regard labor and/or productivity shocks as possible supply shocks; fiscal policy as the source of possible demand shocks; and money supply/demand as candidate nominal shocks. The polynomials in the lag operator, a(L), b(L), and c(L), are assumed to satisfy stationarity conditions so that y_t^s, m_t, and d_t are difference stationary processes. The model predicts the following directional short-run and long-run responses to the supply, demand, and nominal shocks (i.e., $\varepsilon_s, \varepsilon_d$, and ε_n respectively):[1,2]

i) Short Run

	Supply	demand	nominal
Relative real GDP	>0	>0	>0
Real exchange	>0	<0	>0
Relative Price	<0	>0	>0

ii) Long Run

	Supply	demand	nominal
Relative real GDP	>0	0	0
Real exchange	>0	<0	0
Relative Price	<0	>0	>0

3. Identification of the Structural VAR

Let $\Delta x_t = [\Delta y_t \;\; \Delta q_t \;\; \Delta p_t]^T$ denote the system's three variables and $\varepsilon_t = [\varepsilon_{st} \;\; \varepsilon_{dt} \;\; \varepsilon_{nt}]^T$ denote the system's three structural disturbances. I assume that $\Delta x_t = [\Delta y_t \;\; \Delta q_t \;\; \Delta p_t]^T$ are generated by the following structural moving average (MA) model.

$$\Delta x_t = c + C_0\varepsilon_t + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + .. \tag{6}$$

where C_0 is the 3×3 matrix that defines the system's contemporaneous structural relationships among the three variables. When I estimate a VAR, I do not directly recover estimates of the structural moving average model. Rather, I estimate a p-th order VAR as

$$\Delta x_t = a + A_1\Delta x_{t-1} + A_2\Delta x_{t-2} + .. + A_p x_{t-p} + e_t \tag{7}$$

and then the moving average model

$$\Delta x_t = c + R_0 e_t + R_1 e_{t-1} + R_2 e_{t-2} + .. \tag{8}$$

where $c = A(1)^{-1}a$, $R_0 = I$, and $R_k = \sum_{j=1}^{k} A_j R_{k-j}$ (9)

and R_k denotes the matrix of coefficients for the k-th moving average term after estimating the VAR.

The summation of the vector moving average coefficients as the lags go to infinity $(VMA(\infty))$ can be approximated through

$$R(1) = \sum_{j=1}^{\infty} R_j = [I - A_1 - A_2 - .. - A_p]^{-1} \tag{10}$$

in other words by approximating the infinite order VMA representation by the finite summation of the individual moving average coefficients. Comparing the two representations

$$\Delta x_t = c + C_0\varepsilon_t + C_1\varepsilon_{t-1} + C_2\varepsilon_{t-2} + ..$$

$$\Delta x_t = c + R_0 e_t + R_1 e_{t-1} + R_2 e_{t-2} + ..$$

and assuming that there exists a nonsingular matrix S such that

$$e_t = S\varepsilon_t \tag{11}$$

one can obtain:

$$e_t = C_0\varepsilon_t \tag{12}$$

$C_0 = S$, and $C_k = R_k C_0$.

In addition to recovering estimates of the parameters that define the structural MA in (6), one can also recover an estimate of the symmetric covariance matrix of the reduced form disturbances, Σ and

$$\Sigma = E[e_t e_t^T] \tag{13}$$

Suppose as is common in the literature, that the structural shocks are mutually orthogonal and that each has unit variance. Then, from (12) and (13)

$$C_0 C_0^T = \Sigma \tag{14}$$

This represents a system of six equations in nine unknowns. The six equations are derived from the three variances and three covariances that define Σ. Thus, three additional restrictions are needed to identify C_0 and to recover the time series of structural shocks ε_t, as well as the structural system dynamics defined by $C_1, C_2, \ldots$. My open economy macro model is triangular in the long run. That is only supply shocks ε_{st} are expected to influence relative output levels, while both supply and real aggregate demand shocks ε_{dt} are expected to influence the real exchange rate in the long run. The nominal shocks ε_{nt} have no long run impact on either relative output levels or the real exchange rate. It can be shown that

$$C(1) = C_0 + C_1 + C_2 + .. = R(1)C_0 \tag{15}$$

Using the notation of the model, these restrictions are:

i) neither nominal nor demand shocks affect relative output levels in the long run:

$$C_{12}(1) = C_{13}(1) = 0 \tag{16}$$

ii) nominal shocks do not influence the real exchange rate in the long run:

$$C_{23}(1) = 0 \tag{17}$$

where $C_{ij}(1)$ denotes the infinite sum of the moving average coefficients of shock j on variable i. With the above restrictions $C(1)$ is a lower triangular matrix. The above three restrictions implied by the model exactly identify the structural matrix C_0. For the initial analysis I do not impose the restriction that the nominal shocks have a proportional effect (one for one) on the relative prices.

With this additional restriction of $C_{33}(1)=1$, I would have an over-identified system. I briefly touch on the over-identified system in section 4.5. Re-expressing $R(1)$

$$R(1)=R_0+R_1+R_2+..=(I-\sum_{j=1}^{p}A_j)^{-1} \tag{18}$$

$$\Delta x_t=R(1)e_t=C(1)\varepsilon_t$$

Then,

$$R(1)\Sigma R(1)^T=C(1)C(1)^T \tag{19}$$

$R(1)\Sigma R(1)^T$ is a positive definite symmetric matrix and thus admits a lower triangular Cholesky decomposition. That is how $C(1)$ can be calculated after the VAR estimation and obtaining the infinite order MA representation. The Cholesky decomposition is unique up to a sign transformation of the columns of the matrix. After solving for C(1), the contemporaneous impulse response matrix C_0 can be derived as:

$$C_0=R(1)^{-1}C(1) \tag{20}$$

The matrix C_0 is also unique up to a sign transformation. Let us denote this property by using the previously defined relation $C_0C_0^{\,T}=\Sigma$:

$$\begin{bmatrix} c_{11}(0) & c_{12}(0) & c_{13}(0) \\ c_{21}(0) & c_{22}(0) & c_{23}(0) \\ c_{31}(0) & c_{32}(0) & c_{33}(0) \end{bmatrix}\begin{bmatrix} c_{11}(0) & c_{21}(0) & c_{31}(0) \\ c_{12}(0) & c_{22}(0) & c_{32}(0) \\ c_{13}(0) & c_{23}(0) & c_{33}(0) \end{bmatrix}=\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33} \end{bmatrix}$$

Thus,

$$\begin{aligned}
\sigma_{11}&=c_{11}(0)^2+c_{12}(0)^2+c_{13}(0)^2\\
\sigma_{22}&=c_{21}(0)^2+c_{22}(0)^2+c_{23}(0)^2\\
\sigma_{33}&=c_{31}(0)^2+c_{32}(0)^2+c_{33}(0)^2\\
\sigma_{12}&=c_{11}(0)c_{21}(0)+c_{12}(0)c_{22}(0)+c_{13}(0)c_{23}(0)\\
\sigma_{13}&=c_{11}(0)c_{31}(0)+c_{12}(0)c_{32}(0)+c_{13}(0)c_{33}(0)\\
\sigma_{23}&=c_{21}(0)c_{31}(0)+c_{22}(0)c_{32}(0)+c_{23}(0)c_{33}(0)
\end{aligned} \tag{21}$$

The crucial point to recognize here is that the groups of parameters $\{c_{11}(0),c_{21}(0),c_{31}(0)\}$, $\{c_{12}(0),c_{22}(0),c_{32}(0)\}$ and $\{c_{13}(0),c_{23}(0),c_{33}(0)\}$ can change sign together without affecting the equations. The interesting nature of the restrictions lends itself to eight different solutions for the nine parameters, where the absolute values of the solutions are all equal for each parameter, and the plausible solution can be judged using an exchange rate determination model. The way I determine the signs of the elements of C_0 will be by taking into account the long run effects of supply shocks on relative real GDP, long run effect of demands shocks on real exchange rate and long run effect of nominal shocks on relative prices or inflation[3]. After deciding on C_0, the structural dynamics, C_k, can be derived using $C_k = R_k C_0$, where C_k is the impulse response matrix for the k-th lag of the structural disturbances. The accumulated impulse response of a shock at time t of a structural disturbance after k lags is $\sum_{j=0}^{k} C_j$. These are also called the accumulated impulse response functions of a supply, demand or nominal shock on the levels of the variables relative real GDP, real exchange rate and the relative prices.

The forecast error variance decompositions for each variable shows what proportion of the forecast error variance at different forecast horizons can be attributed to each structural shock in the model. For example the forecast error variance of the $i-th$ variable in Δx_t due to the structural disturbance j at the $k-th$ lag will be:

$$\frac{\sum_{m=0}^{k} c_{ij}(m)^2}{\sum_{l=1}^{3}\sum_{m=0}^{k} c_{il}(m)^2} \tag{22}$$

where $c_{ij}(m)$ is the $(i,j)^{th}$ element of the matrix C_m.

One might also be interested in the forecast error variance decomposition of the variables in levels rather than in differences. For instance, the forecast error variance decomposition for the level of relative output would, by construction, force the contribution of supply shocks to asymptote to 100 percent as the forecast horizon lengthened, and the contributions of the other shocks would commensurately decline toward zero.

Let us try to find the forecast error variance at the levels for the short horizons:

Let forecast horizon be zero. Then:

$$\Delta x_t - E_t(\Delta x_t) = C_0 \varepsilon_t$$

$$x_t - x_{t-1} - E_t x_t - E_t x_{t-1} = x_t - E_t x_t = C_0 \varepsilon_t$$

Then, the forecast error for the levels of the variables is $C_0 \varepsilon_t$.

Let the horizon be equal to one. Then:

$$\Delta x_{t+1} - E_t(\Delta x_{t+1}) = C_0\varepsilon_{t+1} + C_1\varepsilon_t$$

$$x_{t+1} - E_t x_{t+1} = C_0\varepsilon_{t+1} + C_1\varepsilon_t + x_t - E_t x_t = C_0\varepsilon_t + C_0\varepsilon_{t+1} + C_1\varepsilon_t$$

For horizon equal to two:

$$x_{t+2} - E_t x_{t+2} = C_0\varepsilon_{t+2} + [C_0 + C_1]\varepsilon_{t+1} + [C_0 + C_1 + C_2]\varepsilon_t$$

Let us denote the accumulated impulse response at the k^{th} horizon by AIMP(k)

Expressing the forecast error at horizon 0,1,2:

Horizon	Forecast error	
0	AIMP(0)	
1	AIMP(0)+AIMP(1)	(23)
2	AIMP(0)+AIMP(1)+AIMP(2)	

$$AIMP(0) = C_0,\ AIMP(1) = C_0 + C_1,\ AIMP(2) = C_0 + C_1 + C_2$$

Accordingly, the forecast error variance of the $i-th$ variable in x_t due to the structural disturbance j for the forecast horizon k will be:

$$\frac{\sum_{m=0}^{k} aimp_{ij}(m)^2}{\sum_{l=1}^{3}\sum_{m=0}^{k} aimp_{il}(m)^2} \tag{24}$$

where $aimp_{ij}(m)$ is the $(i,j)^{th}$ element of the matrix $AIMP(m)$.

4. Estimation

4.1. Data

I use quarterly observations for the G-7 countries (with the U.S. as the home country) from 1973:II – 1992:IV, except for Germany for which the data spans 1978:III – 1992:IV. The data samples roughly correspond to the samples used by Clarida and Gali (1995) and I have expanded the set of countries by adding France and Italy.[4] The transformed variables are: y, the log of real GDP, U.S. less foreign; q, the log of the real exchange rate, the price of foreign goods expressed in terms of U.S. goods; and, p, the log of the relative price level, measured by the Consumer Price Index (CPI), U.S. less foreign.

The VAR used by Clarida and Gali (1995) is expressed in terms of the first differences of relative real GDP (Δy_t), the real exchange rate (Δq_t), and the relative prices level (Δp_t).

Therefore, it is assumed that y_t, q_t, and p_t are unit root processes, but they are not cointegrated. Unit root test results are reported in Table 1. The Phillips-Perron (PP) test fails to reject the unit root null hypothesis for the real exchange rate for any of the six countries. The Augmented Dickey-Fuller (ADF) test rejects the unit root in the real exchange rate for only one country, the United Kingdom. ADF and PP tests of the unit root in the relative real GDP series do not reject the null against the alternative of trend-stationarity at the 5% significance level, except for real GDP relative to Canada where both tests reject the unit root null against the trend-stationarity alternative. The unit root tests for the relative prices reject the unit root null against the trend-stationary alternative for Japan and render conflicting results for the United Kingdom.

For the four cases where one or both tests reject the unit root null at the five-percent level, Kwiatkowski, et al (1992, KPSS) tests of the (trend) stationarity null were applied and the results are reported in Table 2. For U.S. real GDP relative to Canada and the U.S. price level relative to the U.K., the KPSS test rejects the trend-stationary null at the five-percent level. However, the KPSS test does not reject the trend-stationary null for the U.S. price level relative to Japan and it does not reject the stationarity null for the U.S.-U.K. real exchange rate.

I also performed unit root and stationarity tests for the differenced version of the series. The results, which are available upon request, indicate that each series appears to have at most one unit root. Thus, although there are some exceptions, my results suggest that it is reasonable to proceed as in Clarida and Gali (1995) and treat all variables as having a single unit root, achieving stationarity by first-differencing.[5]

Johansen tests for the number of cointegrating vectors in the system $\{y, q, p\}$ are contained in Table 3. The likelihood ratio test (LR) was applied to the trivariate VAR in levels as a starting point to help determine the lag lengths for the cointegration tests. Letting r denote the rank of the cointegrating space, the λ_{max} tests applied are of the form, H_0: $r = 0$ vs. H_A: $r = 1$. The λ_{trace} tests are of the form, H_0: $r = 0$ vs. H_A: $r > 0$. Once small sample corrections (Cheung and Lai, 1993) are applied to the cointegration test statistics λ_{trace} and λ_{max}, I find no evidence of cointegration at the five-percent significance level using the λ_{max} test, though the five-percent sized λ_{trace} tests suggest there is at least one cointegration vector for France and Italy. Overall, however, these results provide a rationale to proceed as in Clarida and Gali (1995) and assume the existence of finite-order VARs in first differences without including any error-correction terms.

The VAR was estimated by OLS for each country using the same lag lengths as in Table 3. The estimated VAR and the identifying restrictions were applied as in Blanchard and Quah (1989) to estimate the VMA, which was then used to construct impulse response functions and variance decomposition tables.

4.2. Impulse Response Functions

Figures 1.a, 1.b, and 1.c illustrate the estimated impulse response functions along with 90-percent confidence bands. The lag lengths are the same as those used for the cointegration tests reported in Table 3. Kilian (1998) shows that constructing confidence bands for impulse response functions in VARs using the asymptotically correct normal approximation can be misleading in small samples. Table 4 shows the results from applying Kolomogorov-Smirnov tests for normality based upon bootstrap samples of a set of impulse responses for the U.S.-Canadian real exchange rate. The bootstrap distribution of the impulse responses is skewed and normality is rejected. Therefore, I use Runkle's (1987) bootstrap procedure in the construction of the confidence bands.[6]

Consider first the point estimates of the impulse responses. I want to examine whether the signs of the short- and long-run responses of U.S. relative output, the U.S. real exchange rate, and the U.S. relative price level to positive one-unit relative supply, demand, and nominal shocks are consistent with the predictions of the structural model. Or, as Clarida and Gali (1995) put it, I use the estimated impulse responses "to assess whether or not the shocks that my procedure identifies as supply, demand, and nominal shocks 'look like' supply, demand, and nominal shocks are supposed to look."

In support of the model, note that for all six countries: relative real GDP increases in response to a relative supply shock, the relative price level increases after a positive nominal shock, and the real exchange rate appreciates after a positive demand shock. These responses are consistent with the theory for both the short-run and the long run.

However, there are many discrepancies between the estimated responses and the predictions of the theory. Positive "demand" shocks are found to decrease U.S. output levels relative to France, Germany and Italy and they are found to decrease U.S. prices in the long-run relative to France, Germany, Italy, Japan, and the United Kingdom. Positive "nominal" shocks are found to decrease U.S. output in the short-run relative to Japan and initially cause the French real exchange rate to appreciate. Positive "supply" shocks are found to decrease the real exchange rate for Germany, France, Italy and the United Kingdom and these shocks are found to increase the long-run price level relative to Japan and Germany. Thus, I find numerous discrepancies between the estimated responses and the theory's predictions. The Canadian case is the only one that exhibits the predicted responses to all three shocks.

Of course, it can be argued that point estimates only tell part of the story and that sampling error should be taken into account through the construction of confidence intervals. However, in this case it appears that the data are of little use in helping us understand real exchange rate determination through the use of the structural VARs. In particular, observe in Figures 1.a, 1.b, and 1.c that the confidence intervals are consistent with the hypotheses that: supply shocks and nominal shocks do not have significant effects on the real exchange rate for any of the six countries; supply shocks do not have significant effects on the relative price level for any country; and, most of the responses to demand and nominal shocks are not significant. [7]

4.3. Robustness: estimating with different lag lengths

There are different ways one can handle the choice of lag lengths in a VAR framework. Among the alternative procedures commonly used are the Schwartz Bayesian Criteria (SBC), the Akaike Information Criteria (AIC), and the Likelihood Ratio Test (LR). In this section, I consider how robust the structural VAR results are to different lag lengths.

The results presented above were based on the LR test combined with an examination of the properties of the residual.[8] In four cases (Germany, Italy, Japan, and the U.K.) my criteria selected the lag length of four, coinciding with the lag length selected without pretests by Clarida and Gali (1995). However, for Canada and France, the criteria selected a lag length of three. Figure 2 illustrates how the results for Canada and France differ according to whether the lag length is set at three or four, presenting us with a glimpse into the sensitivity of the full set of results to lag length variation. For Canada, the real exchange rate impulse responses to a nominal shock with a VAR lag of three in differences (and, therefore, four in levels) are in accord with the theory. However, with a lag length of four in differences, the short-run sign is reversed with the real exchange rate initially appreciating, then converging to zero without ever depreciating. For France, the LR-selected lag of three generates an unexpected exchange rate appreciation in the short run. However, if I increase the number of lags to four, I observe the expected short-run behavior in the exchange rate.

4.4. Canada and cointegration

Canada is the only country in my set for which the point estimates of the impulse responses have the correct signs both in the short and long run (Figure 1a). Moreover, in this case, the real exchange rate depreciates and overshoots in response to a nominal shock. Overall, Canada seems to be the best candidate for satisfying the model's predictions. This brings up the question: in what way is Canada different from the other five countries in the sample?

Table 3 gives a possible explanation. The existence of a finite-order VAR in first-differences relies on the no-cointegration assumption. Previously, I argued that this assumption can be justified for all six countries if the $\lambda_{\max}$ test is used with a five-percent significance level and if the Cheung and Lai (1993) small sample correction is applied. However, if the λ_{trace} test is used, or if the significance level is increased to 10-percent, or if asymptotic critical values are used, the argument for no cointegration weakens considerably. In fact, Canada turns out to be the only country for which the null of no cointegration cannot be rejected regardless of which test statistic is applied, whether the five-percent or 10-percent significance level is used, and whether a small sample correction is or is not made. Put another way, for every other country in the sample an argument can be made that cointegration appears to be present and so the fundamental assumption of a finite-order VAR in first-differences is inappropriate and the results that follow may be misleading.

To push this point a bit further, consider Figure 3, which presents the impulse response functions of the real exchange rate due to a supply shock for Canada using two different data samples: the original sample, 1973:II – 1992:IV, and an extended sample 1973:II – 1997:III. The sign of the long-run response of the exchange rate to a positive supply shock depends on the sample period. The U.S.-Canada real exchange rate depreciates when I use my original sample, but it appreciates when I use the extended sample. What is striking is that the sign reversal is related to the results of cointegration tests. When the λ_{trace} statistics indicates possible cointegration at the five-percent or 10-percent level, the sign is reversed and the theoretically incorrect appreciation of the real exchange rate is observed for Canada.

4.5. Over-identifying Restrictions

In Clarida and Gali model one unit nominal shock has a one-for-one effect on the relative prices in the long run. My initial decompositions do not take it into account as the model is already exactly identified by the three long-run restrictions. The additional restriction for the nominal shock is an over-identifying restriction and I check for the validity of the restriction in the model through χ^2 tests. The test statistics in Table 5 strongly reject the over-identifying restrictions and therefore my decomposition is justified. This is not surprising given the shortcomings of a structural VAR model. In my decomposition all temporary shocks are aggregated under the name nominal shock. Although I expect that shocks to money supply and money demand are predominant among the temporary shocks, it is not likely that all transitory shocks to the system will create the aggregate effect on the price level bringing about one-for-one rise due to a nominal shock.

4.6. Variance decompositions

Table 5.a provides decompositions of the k-step-ahead error variance in forecasting the (logged) level of the real exchange rate into the proportion attributable to each of the structural innovations. Under the assumption that the real exchange rate is a unit root process, the unconditional variance of the real exchange, which is the limit of the k-step forecast error variance as k goes to infinity, is not finite. However, the unconditional variance of the stationary first-differenced exchange rate is finite. Its decomposition can be inferred from Table 5.b, which provides decompositions of the conditional variance of the differenced log of the real exchange rate into the proportion attributable to each of the structural innovations.

Table 6 also includes 90-percent confidence intervals for each estimated proportion. Lutkepohl (1990) shows that, if the restriction that they sum to unity is not imposed, the estimated proportions are asymptotically normal. However, as Figure 4 illustrates for the Canadian case, the bootstrap distributions of the variance decompositions are rather skewed. For example, the proportions of the forecast error variance in the real exchange rate and in the relative price level due to supply shocks are strongly skewed to the left. However, the proportion of the forecast error variance in relative real GDP due to supply shocks is strongly skewed to the right. As a result, I chose to report 90-percent confidence intervals derived from the empirical bootstrap distribution.

First, consider the conditional variances of the (logged level of the) real exchange rate reported in Table 6.a. For all six countries, demand shocks dominate in the short-run and long-run. For Canada and the United Kingdom, demand shocks account for over 90-percent of the forecast error variance at horizons 1, 4, 12, and 24 quarters. For Germany, demand shocks account for over 90-percent of the forecast error variance at horizons 1, 4, and 12 and they account for 86-percent of the conditional variance at horizon 24. For France, demand shocks account for 90-percent of the forecast error variance at horizon 1, with this proportion declining monotonically to 80-percent as the horizon increases to 24. For Japan, demand shocks account for 72-percent of the conditional variance one-quarter ahead, monotonically increasing to 94-percent at 24 quarters ahead. Demand shocks account for 62-percent to 73-percent of the conditional variance for Italy. Notice the size of the corresponding 90-percent confidence intervals. If I use the lower ends of these intervals (and upper ends of the intervals for the other shocks) I still conclude that demand shocks are an important source of variation in the exchange rate, though they become less important than supply shocks for France, Germany, Japan, and especially, Italy.

Nominal shocks explain less than 10-percent of the conditional variance in the real exchange rate, regardless of the country or horizon. For the France, Italy, and the U.K., the proportion is less than one-percent at all horizons. For Germany and Japan, it is less than four-percent at all horizons. However, if I use the upper ends of the 90-percent confidence intervals, the story changes substantially. For example, at one-quarter ahead, nominal shocks explain about 20-percent of the conditional variance in the Japanese real exchange rate, 30-percent for France, 35-percent for Canada, 43-percent for the United Kingdom, 50-percent for Italy, and 57-percent for Germany. As the horizon increases, the upper ends of these intervals decrease, and in all six cases they fall below 10-percent by horizon 24. So, although there appears to be a lot of uncertainty about the short-run importance of nominal shocks on the real exchange rate, the evidence for long-run neutrality is much stronger.

Supply shocks account for at least 25-percent of the conditional variance of the Italian exchange rate and 10-percent of the French exchange rate at horizons 1, 4,12, and 24, becoming more important as the horizon increases for both countries. These shocks play an important role in explaining the short-run (i.e., one-quarter and four quarters ahead) real exchange rate for Japan and the long-run (i.e., 24-quarters ahead) real exchange rate for Germany. They explain less than 10-percent of the conditional variance in the exchange rates of Canada and the U.K. at all horizons. However, the 90-percenet confidence intervals indicate that these conclusions are very tenuous. For example, using the upper ends of these intervals, observe that supply shocks may explain as much as 40-percent to 50-percent of the exchange rate for Canada at all horizons. Using the lower ends of these intervals, observe that supply shocks may explain as little as two-percent of the exchange rate for Italy at all horizons.

Next, consider Table 6.b, which presents the forecast-error variance decompositions for the differenced logs of the real exchange rate. The proportion of the k-step-ahead forecast-error variance in the differenced exchange rate attributable to a particular type of shock will converge as k increases to the proportion of the unconditional variance attributable to that shock. In Table 5.b I observe that convergence occurs within the first 12 quarters. I also observe that demand shocks account for nearly 90-percent of the variance in the differenced exchange rate for Canada, France, Germany and the U.K., and about 70-percent of the variance for Italy and Japan. Supply shocks play an important role in explaining the variance of the differenced exchange rate for Italy and Japan, explaining about 25-percent of the unconditional variances. Nominal shocks explain between three-percent and nine-percent of the unconditional variances in these two cases. If I consider the implications of the confidence intervals, very different conclusions can be drawn. For example, using the upper ends of these intervals, nominal shocks can explain about 25-percent of the variance in the Japanese rate, about 30-percent to 40-percent of the variance in the Canadian and French rates, and about 50-percent of the variance in the German, Italian, and U.K. rates. If I use the lower ends of these intervals, supply shocks explain less than 10-percent of the exchange rate variance for all six countries.

My third-order lag length selections for Canada and France differ from Clarida and Gali's (1994) fourth-order specification for all countries in their sample. Table 6.c provides results when I re-estimate the models for Canada and France using fourth-order VARs. The left-hand-side of Table 5.c presents the decompositions of the k-step-ahead forecast error variance for the logged levels of the real exchange rate, while the right-hand-side of Table 5.c presents the decompositions of the k-step-ahead forecast error variance for the differenced logs of the real exchange rate. Observe that i) supply shocks become more important for both countries as the lag length is increased from three to four; ii) demand shocks remain the dominant source of variation in both exchange rates, though their importance drops somewhat; and, iii) nominal shocks continue to be relatively unimportant.

5. Conclusion

This paper has provided new evidence on the difficulty of drawing conclusions about real exchange rate determination based upon structural VARs. The problems I address, such as the sensitivity of the results to lag length selection, are well-known pitfalls of structural VAR analysis. My contribution is to illustrate the consequences of these problems for the specific issue of real exchange rate determination, focussing on the Clarida and Gali (1995) paper. My focus is on Clarida and Gali (1995) because I think that, despite the concerns I raise, it is correctly recognized to be a very important paper in the empirical exchange rate literature.

Using the structural VAR model of Clarida and Gali (1995), but with slightly different data and an expanded set of countries, I tried to identify supply, demand, and monetary shocks using estimated impulse responses and the predictions of the structural model. In contrast to their findings, I conclude that the impulse responses do not easily lend themselves to the interpretations as being responses to the supply, demand, and nominal shocks that are part of the structural model. I find perverse signs for the impulse responses and, based upon bootstrap confidence intervals, these responses are insignificant for most cases and reject any overshooting of real exchange rate due to nominal shocks. Setting aside the question of how to interpret the shocks identified by the empirical procedure, point estimates of the proportions of the k-step-ahead forecast error variance in the real exchange rate attributable to each of the three shocks provide a reasonably consistent story across the six countries I considered. Demand shocks play the most important role, supply shocks play a substantial role, and nominal shocks are inconsequential. However, the confidence intervals for these estimates tend to be so large, that very different stories are also consistent with these estimates. Other problems I identified include sensitivity of key results to lag length selection and the strong possibility that there are cointegrating relationships that should be accounted for and are distorting the results.

I conclude that only for the case of Canada as the foreign country does the structural VAR approach provides results that can be interpreted within the context of the theoretical exchange rate model. For each of the other countries (France, Germany, Italy, Japan, United Kingdom) I conclude that either the theoretical model is inappropriate or the problems in applying structural VAR methods are too severe to provide meaningful results in this application.

Notes

1. Derivations for the flexible and sticky price equilibrium can be found in Clarida and Gali (1995). It may be argued that demand disturbances have a long-run impact on output. Following Blanchard and Quah (1989), I assume that such long-run effects are small compared to those of supply disturbances and thus my decomposition approaches the correct identification.
2. Clarida and Gali (1995) model the supply and monetary shocks as random walks. They model the differenced demand shock as an MA(1) process with an MA coefficient between zero and one, so that the demand shock has a transitory component. The directional responses I provide are based on those assumptions.
3. Clarida and Gali (1995) and Blanchard and Quah (1989) offer a rigorous explanation of the decomposition procedure, which identifies $C(1)$ up to a sign of its diagonal elements. I resolve these ambiguities by trying to match the predictions for certain long-run effects of the structural shocks with the estimated long-run impulse responses. These long-run selection criteria are: supply shocks have positive effects on GDP, demand shocks appreciate the real exchange rate, and nominal shocks increase the price level.
4. My data for Canada and Germany are from the International Financial Statistics (IFS) March 1998 CD-ROM. The IFS data tapes had data for Germany starting from the third quarter of 1978. Data for the other countries are from the IFS March 2000 CD-ROM. Clarida and Gali (1995) use quarterly data 1976:III – 1992:IV for Japan, 1974:III – 1992:IV for Canada, Britain and Germany. Their data sources are as follows: CPI data are from International Financial Statistics (IFS) data tapes, real GDP data are from OECD Main Economic Indicators and exchange rate data are from Federal Reserve Bank of New York.
5. My purpose in this paper is to question the conclusiveness of the results presented in this line of literature. To the extent that the unit root assumptions are incorrect or there are cointegrating relationships not accounted for, my argument is strengthened.
6. All bootstrap results reported here are based on 1000 bootstrap samples. Kilian (1998) recommends a bootstrap-after-bootstrap method to get more reliable confidence bands for impulse responses, finding that standard bootstrap procedures do not work very well for longer-run responses. My interest is primarily on short-run responses. I constructed

confidence intervals through Monte-Carlo integration procedure, as well, which is the second best in Kilian's analysis. The conclusions do not change regardless of which procedure is used.

7. Note that standard bootstrap intervals do not guarantee that the point estimates will be within the confidence bands. The point estimates of the relative price responses to a nominal shock for Italy are outside the 90% confidence interval. This is probably due to the extreme skewness of the bootstrap distribution of these responses.
8. I checked the autocorrelation of the residuals in the VAR estimation in levels using Ljung-Box test statistics and increased the LR-based lag length until I was satisfied that the residual behavior was consistent with white noise errors. Had I applied the AIC to select lag length I would have chosen lag lengths of two (Germany), four (Canada, Italy), eight (France), 11 (Japan), and 12 (U.K.).

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TABLE 1

UNIT ROOT TEST RESULTS

Variable		Canada	Japan	Country U.K.	Germany	France	Italy
ADF Tests:							
y	τ_t	-4.01**	-2.66	-2.45	-0.76	-2.59	-2.24
	τ_μ	-3.26**	-2.55	-1.17	-0.56	-1.43	-2.27
q	τ_μ	-2.24	-1.55	-3.09**	-2.56	-1.86	-1.54
p	τ_t	-2.78	-4.41**	-3.90**	-0.19	-1.46	0.42
	τ_μ	-1.83	-1.58	-3.14**	-2.43	1.72	-2.03
PP Tests:							
y	τ_t	-3.46**	-2.20	-2.49	-1.06	-2.50	-3.17*
	τ_μ	-2.09	-1.97	-1.12	-1.96	-1.60	-1.40
q	τ_μ	-1.34	-1.42	-2.03	-1.80	-1.91	-1.03
p	τ_t	-1.84	-4.23**	-2.30	0.32	0.16	0.14
	τ_μ	-1.34	0.32	-3.68**	-3.01**	-2.24	-3.14**

y = relative logged real GDP, q = logged real exchange rate, p = relative logged price levels.

ADF: Augmented Dickey-Fuller test statistics. The number of lags included in the ADF regressions were selected by the AIC. PP: Phillips-Perron test statistics, constructed using the first four autocovariances. Critical values from the Dickey-Fuller distributions with sample size 100 were used for all countries, except for Germany where the critical values for sample size 50 were used. * denotes significant at the 10% level and ** denotes significant at the 5% level.

TABLE 2

KPSS TESTS FOR (TREND) STATIONARITY

Country	Variable	Test Statistic
Canada	y	0.148**
U.K.	p	0.183**
Japan	p	0.099
U.K.	q	0.192

y = relative logged real GDP, p = relative logged price level, q = real exchange rate.

The null of trend stationarity and test statistic η_τ were used for relative GDPs and relative price levels. The null of stationarity and test statistic η_μ were used for the real exchange rate. Lag truncation was chosen to be eight as suggested by Kwiatkowski et al (1992). * denotes significance at the 10% level and ** denotes significance at the 5% level, based upon the critical values provided in Kwiatkowski, et al (1992).

TABLE 3

JOHANSEN COINTEGRATION TESTS

(H_0: the cointegrating rank is zero)

Countries	Lags	λ_{max}	λ_{trace}	Small Sample Correction: λ_{max}	Small Sample Correction: λ_{trace}
CANADA	3	13.95	25.77	11.30	20.88
JAPAN	4	19.97*	26.65	16.18	21.59
UK	4	23.66**	33.24**	19.17*	26.93*
GERMANY	4	17.09	31.67**	12.67	23.48
FRANCE	3	24.22**	43.28**	20.54*	36.71**
ITALY	4	18.14	38.54**	14.70	31.22**

For the λ_{max} tests, H_A: cointegrating rank = 1. For the λ_{trace} tests, H_A: cointegrating rank > 0.

The small sample correction applied to the Johansen (1991) statistics, replace T by $T-nm$, where T is the number of observations, n is the number of variables in VAR system, and m is the number of lags. Critical values are from Osterweld-Lenum (1992). Lag is the lag length in VAR with differenced series. Superscripts * and ** denote significance at the 10%, and 5%levels, respectively. The lag lengths were determined by first using the likelihood ratio test and then increasing the lag length until residual tests confirmed no autocorrelation for the first 4 lags and at most 5% for the remaining lags in the residual series for VAR in levels.

TABLE 4

Kolmogorov-Smirnov Tests for Normality of U.S - Canadian Real Exchange Rate Responses

Shock	Horizon	P-Value of K-S Test Statisitc
Supply	4	5.00×10^{-1}
	12	1.21×10^{-3}
	24	5.43×10^{-5}
Demand	4	9.71×10^{-2}
	12	1.07×10^{-15}
	24	1.91×10^{-24}
Nominal	4	1.39×10^{-11}
	12	3.96×10^{-68}
	24	8.99×10^{-236}

VAR lag length = 3

TABLE 5

Over-identification Tests for the Nominal Shock

Countries	χ^2
Canada	784.162
Japan	694.831
United Kingdom	656.555
Germany	531.957
Italy	580.866

The model is exactly identified with three restrictions. The proportional effect (one-for-one) of nominal shock on relative prices adds another restrictions making the model over-identified. Chi-square test has one degree of freedom. The values all have very small p-values.

TABLE 6
DECOMPOSITIONS OF K-STEP-AHEAD FORECAST ERROR VARIANCE IN REAL EXCHANGE RATES ATTRIBUTABLE TO EACH TYPE OF SHOCK WITH 90-PERCENT BOOTSTRAP CONFIDENCE INTERVALS

Part a: Exchange Rate Levels

U.S. – Canada (Lag Length = 3)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.000 (0.001,0.388)	0.909 (0.450,0.976)	0.090 (0.003,0.354)
4	0.016 (0.004,0.406)	0.920 (0.498,0.977)	0.063 (0.003,0.221)
12	0.048 (0.004,0.0480)	0.932 (0.499,0.987)	0.018 (0.001,0.066)
24	0.060 (0.003,0.502)	0.932 (0.491,0.991)	0.008 (0.001,0.034)

U.S. – France (Lag Length =3)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.093 (0.003,0.461)	0.904 (0.429,0.978)	0.003 (0.002,0.304)
4	0.126 (0.009,0.523)	0.869 (0.426,0.974)	0.004 (0.004,0.198)
12	0.177 (0.007,0.584)	0.821 (0.375,0.981)	0.003 (0.002,0.097)
24	0.199 (0.006,0.640)	0.800 (0.357,0.987)	0.001 (0.001,0.054)

U.S. – Germany (Lag Length = 4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.002 (0.004,0.564)	0.960 (0.234,0.966)	0.038 (0.002,0.576)
4	0.015 (0.010,0.574)	0.970 (0.312,0.956)	0.016 (0.007,0.381)
12	0.086 (0.017,0.690)	0.909 (0.272,0.964)	0.005 (0.003,0.156)
24	0.137 (0.013,0.748)	0.860 (0.223,0.976)	0.003 (0.002,0.078)

U.S. – Italy (Lag Length = 4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.264 (0.009,0.724)	0.729 (0.160,0.954)	0.007 (0.002,0.494)
4	0.337 (0.020,0.766)	0.657 (0.167,0.945)	0.006 (0.005,0.340)
12	0.371 (0.020,0.815)	0.626 (0.168,0.959)	0.002 (0.002,0.178)
24	0.380 (0.020,0.828)	0.619 (0.166,0.966)	0.001 (0.001,0.096)

U.S. – Japan (Lag Length = 4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.264 (0.013,0.676)	0.724 (0.278,0.951)	0.021 (0.002,0.209)
4	0.156 (0.024,0.584)	0.812 (0.379,0.947)	0.031 (0.003,0.173)
12	0.079 (0.020,0.525)	0.905 (0.453,0.964)	0.016 (0.002,0.089)
24	0.051 (0.014,0.507)	0.940 (0.485,0.976)	0.009 (0.001,0.053)

U.S. – U.K. (Lag Length =4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.008 (0.004,0.303)	0.987 (0.444,0.978)	0.005 (0.002,0.434)
4	0.025 (0.009,0.358)	0.969 (0.489,0.966)	0.006 (0.006,0.310)
12	0.039 (0.008,0.426)	0.957 (0.524,0.978)	0.004 (0.003,0.141)
24	0.041 (0.006,0.454)	0.957 (0.523,0.986)	0.002 (0.001,0.072)

Table 6, Part b: Exchange Rate Differences

U.S. – Canada (Lag Length = 3)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.000 (0.004,0.380)	0.998 (0.445,0.966)	0.091 (0.006,0.365)
4	0.029 (0.021,0.394)	0.895 (0.458,0.929)	0.076 (0.017,0.297)
12	0.039 (0.024,0.390)	0.877 (0.443,0.921)	0.083 (0.019,0.317)
24	0.039 (0.024,0.400)	0.877 (0.442,0.921)	0.083 (0.019,0.316)

U.S. – France (Lag Length =3)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.084 (0.008,0.435)	0.913 (0.437,0.962)	0.003 (0.005,0.350)
4	0.084 (0.026,0.415)	0.888 (0.426,0.910)	0.028 (0.021,0.369)
12	0.087 (0.030,0.417)	0.883 (0.414,0.904)	0.031 (0.025,0.382)
24	0.087 (0.030,0.420)	0.883 (0.414,0.903)	0.031 (0.025,0.387)

U.S. – Germany (Lag Length = 4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.0021 (0.007,0.553)	0.934 (0.204,0.938)	0.064 (0.005,0.621)
4	0.012 (0.037,0.514)	0.904 (0.217,0.872)	0.084 (0.032,0.607)
12	0.034 (0.063,0.519)	0.879 (0.217.0.830)	0.005 (0.041,0.556)
24	0.036 (0.065,0.521)	0.877 (0.212,0.830)	0.086 (0.042,0.554)

U.S. – Italy (Lag Length = 4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.242 (0.013,-.692)	0.751 (0.161,0.935)	0.007 (0.005,0.496)
4	0.246 (0.053,0.651)	0.727 (0.170,0.876)	0.027 (0.018,0.490)
12	0.244 (0.057,0.642)	0.725 (0.175,0.860)	0.031 (0.027,0.475)
24	0.244 (0.057,0.641)	0.725 (0.178,0.859)	0.031 (0.027,0.474)

U.S. – Japan (Lag Length = 4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.277 (0.036,0.674)	0.701 (0.276,0.920)	0.021 (0.004,0.218)
4	0.281 (0.097,0.631)	0.684 (0.299,0.835)	0.035 (0.017,0.223)
12	0.283 (0.105,0.625)	0.675 (0.296,0.818)	0.042 (0.021,0.243)
24	0.283 (0.104,0.624)	0.674 (0.296,0.818)	0.043 (0.021,0.243)

U.S. – U.K. (Lag Length =4)

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.047 (0.011,0.320)	0.948 (0.005,0.428)	0.004 (0.005,0.428)
4	0.061 (0.035,0.339)	0.889 (0.367,0.886)	0.049 (0.032,0.482)
12	0.066 (0.042,0.346)	0.882 (0.354,0.868)	0.051 (0.040,0.483)
24	0.066 (0.042,0.343)	0.882 (0.354,0.868)	0.051 (0.041,0.483)

Table 6, Part c: Sensitivity to Lag Length Selection

U.S. – Canada, Lag Length = 4

Exchange Rate Levels:

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.034 (0.002,0.402)	0.963 (0.527,0.988)	0.005 (0.001,0.221)
4	0.135 (0.008,0.534)	0.861 (0.420,0.977)	0.004 (0.003,0.136)
12	0.162 (0.008,0.609)	0.836 (0.382,0.985)	0.001 (0.001,0.049)
24	0.166 (0.006,0.632)	0.833 (0.363,0.989)	0.001 (0.001,0.024)

Exchange Rate Differences:

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.033 (0.005,0.393)	0.962 (0.520,0.976)	0.004 (0.002,0.233)
4	0.151 (0.046,0.475)	0.831 (0.423,0.891)	0.017 (0.017,0.229)
12	0.155 (0.058,0.473)	0.818 (0.406,0.871)	0.027 (0.026,0.364)
24	0.155 (0.058,0.474)	0.818 (0.405,0.870)	0.027 (0.026,0.264)

U.S. – France, Lag Length = 4

Exchange Rate Levels:

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shocks
1	0.288 (0.008,0.607)	0.690 (0.252,0.957)	0.022 (0.003,0.505)
4	0.369 (0.017,0.677)	0.617 (0.250,0.904)	0.015 (0.006,0.387)
12	0.393 (0.018.0.732)	0.601 (0.253,0.953)	0.006 (0.004,0.229)
24	0.390 (0.014,0.749)	0.606 (0.240,0.968)	0.003 (0.002,0.150)

Exchange Rate Differences:

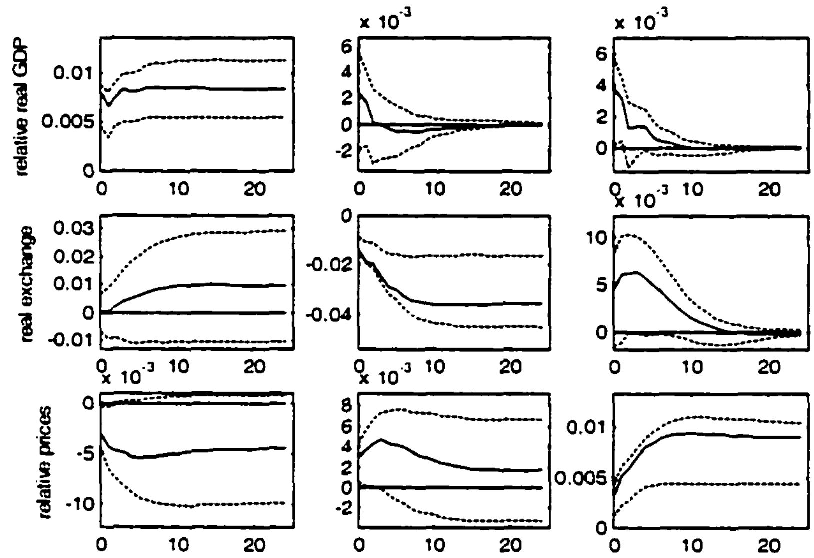
Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock
1	0.249 (0.016,0.550)	0.723 (0.282,0.927)	0.023 (0.007,0.522)
4	0.243 (0.049,0.520)	0.725 (0.282,0.854)	0.032 (0.027,0.486)
12	0.242 (0.060,0.511)	0.720 (0.274,0.826)	0.038 (0.036,0.487)
24	0.242 (0.061,0.511)	0.720 (0.273,0.823)	0.038 (0.036,0.487)

Figure 1.a.

Accumulated Impulse Responses k-Quarters After Shock with 90% Confidence Intervals

Canada (VAR lag = 3)

Shock: Supply Demand Nominal



Japan (VAR lag = 4)

Shock: Supply Demand Nominal

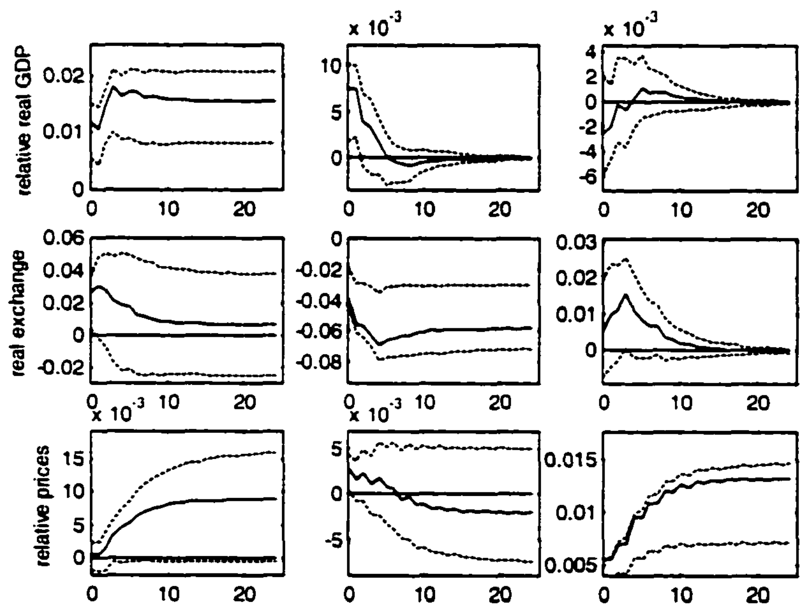
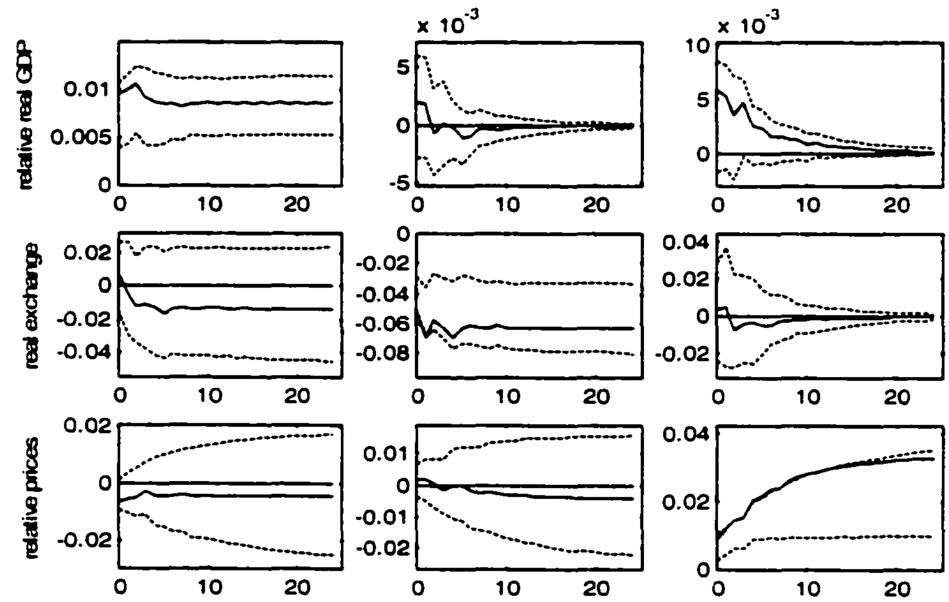


Figure 1.b.

Accumulated Impulse Responses k-Quarters After Shock with 90% Confidence Intervals

United Kingdom (VAR lag = 4)

Shock: Supply Demand Nominal



Germany (VAR lag = 4)

Shock: Supply Demand Nominal

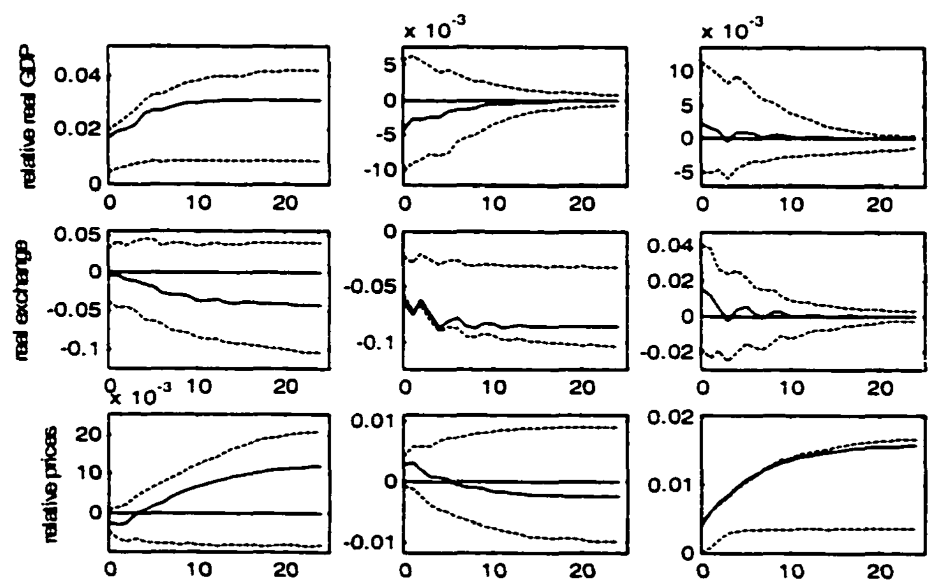
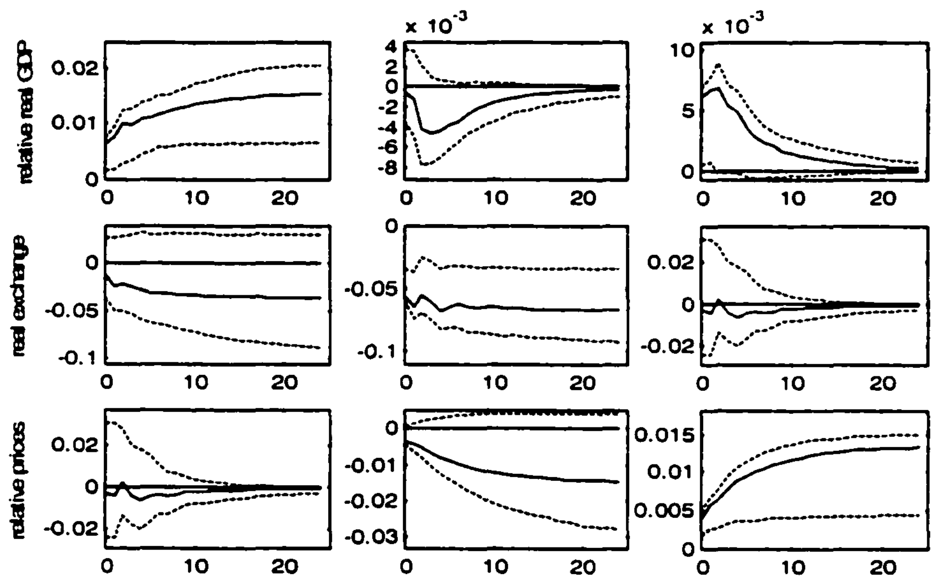


Figure 1.c.

Accumulated Impulse Responses k-Quarters After Shock with 90% Confidence Intervals

France (VAR lag = 3)

Shock: Supply Demand Nominal



Italy (VAR lag = 4)
Shock: Supply Demand Nominal

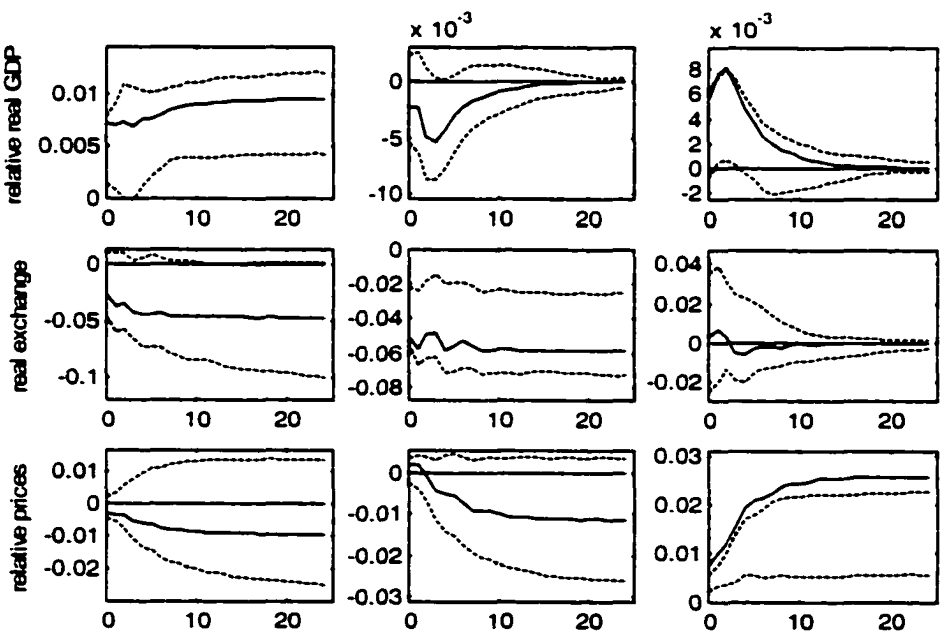
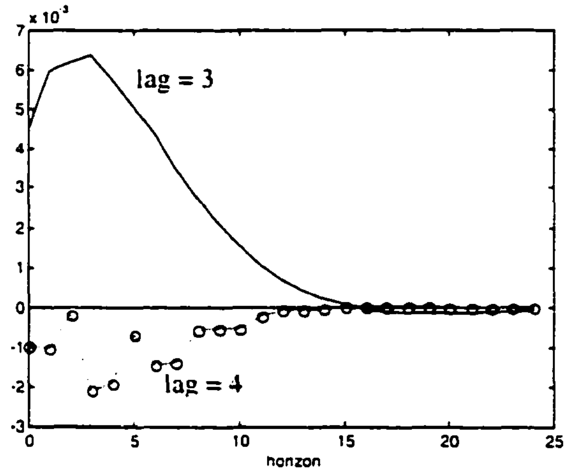


FIGURE 2

Robustness of Impulse Responses to VAR lag specification

Cumulative Response of U.S. Real Exchange Rate to Nominal Shock:

U.S. - Canadian Real Exchange Rate:



U.S. - French Real Exchange Rate:

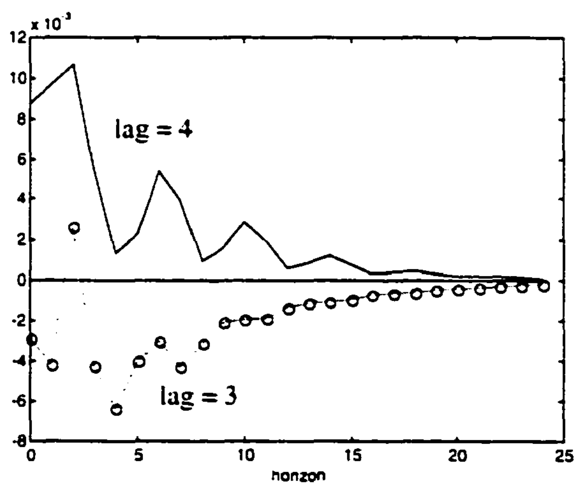
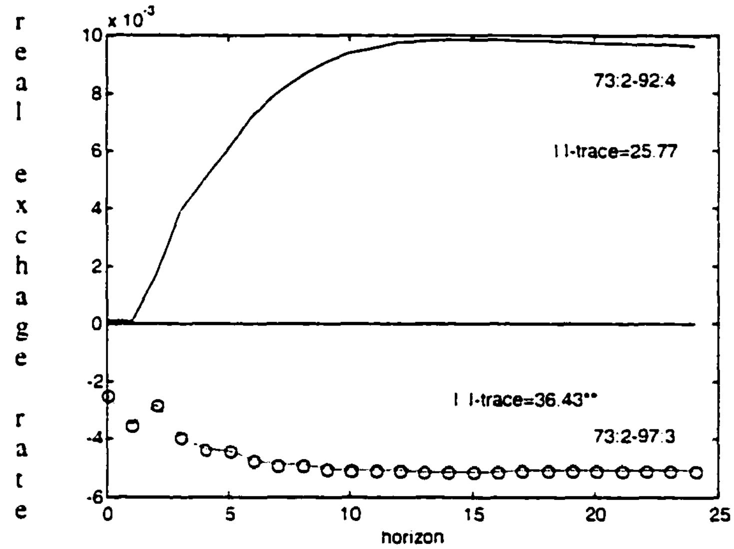


FIGURE 3

Robustness of Impulse Responses to Sample Period

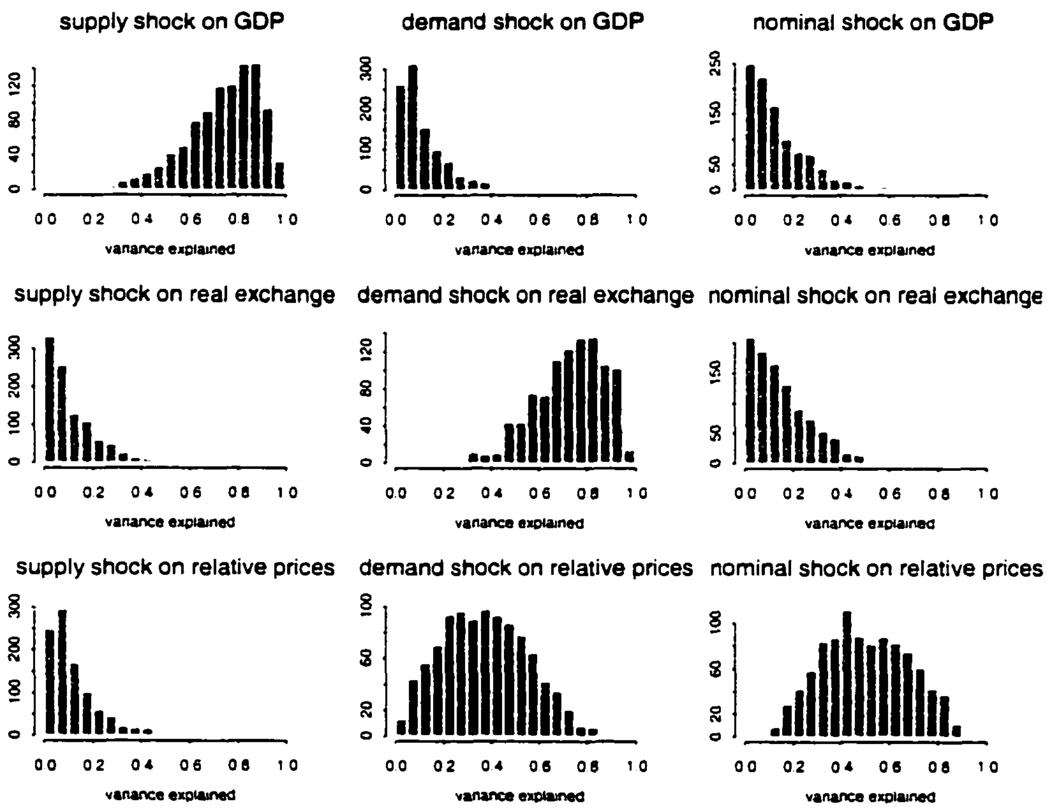
Cumulative Response of U.S. – Canadian Real Exchange Rate to Positive Supply Shocks



VAR lag length = 3
l-trace denotes λ_{trace} test statistic; ** = significant at five-percent level

FIGURE 4

Histograms of Bootstrap Forecast Error Variance Decompositions at 24 Quarter Horizon for Canada



Note: VAR lag length = 3

CHAPTER 3
STRUCTURAL VECTOR ERROR CORRECTION MODELING OF REAL EXCHANGE RATES: CAN WE DO BETTER? EVIDENCE FROM THE G-7 COUNTRIES

1. Introduction

Identifying and measuring the sources of real exchange rate fluctuations has been a serious challenge for empirical macroeconomics. Previous studies of the sources of real exchange rate fluctuations have concluded that real demand shocks account for most of the variance of real exchange rates, in the short-run as well as in the long run (Clarida and Gali, 1995; Weber, 1997; Chadha and Prasad, 1997). Diverging results are presented by Rogers (1999), Eichenbaum and Evans (1995) who document a larger influence of monetary shocks, Alexius (2000) who concludes, in contrast to other findings in the literature, that supply shocks have a larger relative influence, and Karstensen and Hansen (1997) with the claim that monetary shocks are predominant.

Another notable feature of the literature is that most studies model only changes in real exchange rates and the fundamental variables. Some recent exceptions are Karstensen and Hansen (1997), and Alexius (2000). The presence of long run relationships between the levels of the variables is either rejected (as in Clarida and Gali, 1995; and Rogers, 1999) or not investigated (as in Weber, 1997)[1]. In Chapter two I identify strong cointegration relationships between real exchange rate and other variables in Clarida and Gali (CG) model using a slightly different time period in an extended set of countries. There seems to be a gap between the dominant empirical literature and related studies of long run real exchange rate determination. Most papers on the latter field do find long run equilibrium relationships between real exchange rates and, for instance, relative productivity. MacDonald (1998) provides a comprehensive survey.

In this paper, I expand on standard structural vector autoregressions (SVARs) by bringing cointegration effects into the picture in a simple four-variable vector autoregression (VAR) characterized by cointegration. Data on real US output, real exchange rate, real M1 money balances and US treasury-bill interest rates comprise the structural vector error correction (SVEC) model that includes the money demand as the cointegrating vector. My data spans the same time frame in Chapter two and is quite similar to the stylized paper by CG (1995). Using knowledge of cointegration rank and qualitative expectations of the effects of various economic shocks on the variables in line with CG (1995), and Crowder, Hoffman and Rasche (1999), I identify distinct "real" and "nominal" innovations that dictate the long run behaviour of the variables. I also examine the explanatory power of permanent innovations that are orthogonal to each other. One of the permanent shocks displays all the characteristics of a technology or "supply" innovation, while the other is interpretable as a "demand" side impulse. The permanent nominal shock bears the imprint of an innovation in aggregate inflation expectations. The identifying constraints on the permanent shocks are the same as in CG in that nominal shocks do not affect the real output and the real exchange rate in the long run, and demand shocks do not have a long run effect on real output. The results suggest that of the three permanent innovations on real exchange rates, demand shocks dominate at most horizons for G-7 countries.

The permanent supply shock is clearly linked to the technical capacity of the economy governed by productivity innovations, while the permanent demand shock exhibits movements similar to the changes in government expenditures. The permanent nominal innovation appears closely linked with movements in aggregate inflation expectations taken from survey data. Finally, transitory innovations are a "catch-all" term representing all types of temporary innovations on the variables in SVEC system.

I extend the application of Warne (1993) in an environment with emphasis on real exchange rates while confirming CG that demand shocks dominate real exchange rate fluctuations. My decomposition also justifies the concerns raised in Chapter two that non-modeled cointegration effects may be what is driving the poor performance of structural VARs and also what may be behind the commonly identified "perverse sign effect" of supply shock to the real exchange rate in the post-Bretton Woods floating exchange rate period. I examine the robustness of the SVEC model to different lag selection criteria and show that "perverse sign effect" of supply shock may be related to different choices in lags that may create cointegration effects, and hence changes in impulse responses. My analysis reveals both the importance of incorporating cointegration into structural decompositions as well as how closely the stochastic components identified in the process compare with data external to my simple model. The main contribution of the paper is that it provides a SVEC model of real exchange rates in the floating exchange rate period for G-7 countries while retaining the same set of identifying assumptions of CG to identify supply, demand and nominal shocks. My VEC model differs from other studies with the cointegrating vector being defined by an economic behavioral relationship, money demand equation.

In section II, I set up a SVEC model that can be characterized by long run money-demand equation. Shock identification in a cointegration system is illustrated and identifying assumptions for the permanent shocks are explicitly spelled out. Section III provides the empirical analysis for the time series properties of the variables in the VAR system and for the cointegration rank and depicts the estimated parameters for the money demand equation. Section IV presents the impulse responses from the SVEC model of real exchange rates, their variance decompositions and discusses issues related to the robustness of impulse responses to lag length selection. I also compare the identified common stochastic trends with data that are external to the original four-variable specification. Section V summarizes my findings.

2. Identification of Common Trends in a Cointegrated System

Linear time series models are generally specified in terms of variables that can be observed and purely nondeterministic and serially uncorrelated errors. Accordingly, they can be estimated with standard tools. In contrast, a common trends model consists of a vector of trends and a vector of stationary variables, where neither component can be observed as an individual factor. Without loss of generality, let $\{x_t\}$ be a vector of time series such that

$$x_t = x_t^P + x_t^s \tag{1}$$

Here, x_t^P represents a vector of trends of x_t, while x_t^s is a stationary residual.

King Plosser, Stock and Watson (KPSW) (1991) and Stock and Watson (1988) show that there is a simple duality between the concepts of cointegration and common trends. In particular, the cointegrating restrictions determine the number of independent trends and how a vector of observed variables is related to all the independent trends. That is, if β is a cointegrating vector, then $\beta^T x_t^P = 0$ for $\beta^T x_t = \beta^T x_t^s$ to be stationary. These restrictions, however, neither specify nor suggest whether a certain trend is related to, e.g. technology shocks, government expenditure shocks, money shocks. To be able to make such interpretations it is necessary to consider further identifying assumptions. In this section, I devote the first part to the mathematical structure of cointegrated time series with various representations. In the second part I illustrate the restricted VAR representation. The third part lays out the common trends representation. The fourth part identifies permanent and transitory innovations. The fifth part lays out a structural vector error correction model of real exchange rates and the necessary identifying assumptions.

2.1. The Unrestricted VAR and VEC Representation

Let $\{x_t\}$ denote an n dimensional real valued discrete vector time series that is driven by $k \le n$ common stochastic trends. Then, I can write the data generating process (DGP) as:

$$x_t = x_0 + \Psi \tau_t + \Phi(L) v_t \tag{2}$$

Here, L is the lag operator, i.e. $L^j x_t = x_{t-j}$ for any integer j. The n dimensional vector sequence $\{v_t\}$ is assumed to be white noise with $E[v_t]=0$ and $E[v_t v_t^T] = I_n$, the $n \times n$ identity matrix. Furthermore, the $n \times n$ matrix polynomial $\Phi(L) = \sum_{j=1}^{\infty} \Phi_j L^j$ is finite for all L on and inside the unit circle and, without loss of generality, I assume that x_0, the constant vector containing the initial values of x is stationary. In other words, $\Phi(L)v_t$ is jointly (wide-sense) stationary.

The nonstationary (permanent) and stationary (transitory) components of x_t are captured by $\Psi \tau_t$ and $\Phi(L)v_t$, respectively. If the trends are linearly deterministic, then $\tau_t = \mu t$, i.e. $\tau_t - \tau_{t-1} = \mu$, where μ is a k-dimensional vector of constants. The idea of linearly stochastic trends, on the other hand, can be operationalized by modelling τ_t as a vector of random walks with drift:

$$\tau_t = \mu + \tau_{t-1} + \varphi_t \tag{3}$$

where $\{\varphi_t\}$ is a white noise sequence with $E[\varphi_t]=0$ and $E[\varphi_t \varphi_t^T] = I_k$. Hence, φ is a k-dimensional vector of structural (independent) shocks with permanent effects on x if $\Psi \ne 0$. In relation to the decomposition in (1) I find that the common trends model in (2) and (3) specifies that

$$\begin{aligned} x_t^s &= x_0 + \Phi(L)v_t \\ x_t^P &= \Psi\left[\tau_0 + \mu t + \sum_{j=1}^{t} \varphi_j\right] \end{aligned} \tag{4}$$

Furthermore, whenever the number of common trends, k, is less than the number of variables, n, there are exactly $r = n - k$ linearly independent vectors which are orthogonal to the columns of the loading matrix Ψ. In other words, there exists an $n \times r$ matrix β such that $\beta^T x_t^P = 0$ for all t so that $\beta^T x_t$ is jointly stationary. The common trends model in (2) and (3) has some appealing properties. First, the trends include a stochastic element, which is consistent with the notion that some shocks to an economy are persistent. Second, there may be fewer trends than variables so that the model allows for steady state relationships between the variables. In this framework, these steady states are described by the matrix β. If φ_t and v_t are correlated it is possible for the trend disturbances to influence not only growth but also fluctuations about the trends. The approach I shall take in this paper implies that the first k elements of v_t are given by φ_t. To allow for deterministic trend shifts (3) can be reformulated as:

$$\tau_t = \mu + \mu^* D_t + \tau_{t-1} + \varphi_t \tag{5}$$

Here, D_t is a d-dimensional vector of zero-one dummy variables with a finite number of switches. In terms of equation (5), $\mu^* D_t$ may represent both temporary and permanent deterministic changes in the drift component $(\mu + \mu^* D_t)$, i.e. both the 'crash' and 'changing growth' phenomena considered by Perron (1989). The rest of the analysis is based on the reformulation (5).

To determine how I can estimate the common trends model, let us assume that $\{x_t\}$ is generated by the unrestricted vector autoregression (VAR) or order p:

$$A(L)x_t = \rho + \rho^* D_t + \varepsilon_t \tag{6}$$

where $A(L) = I_n - \sum_{j=1}^{p} A_j L^j$, and $\{\varepsilon_t\}$ is a white noise process with $E[\varepsilon_t] = 0$ and $E[\varepsilon_t \varepsilon_t^T] = \Sigma$., a positive definite matrix. The $n \times n$ matrix polynomial $A(L)$ satisfies $\det[A(L)] = 0$ if and only if $|L| > 1$ or $L = 1$ so that explosive processes are ruled out. Moreover, the only form of nonstationarity which is possible is due to unit roots. In other words, if $\{x_t\}$ is generated by (6), then the process is integrated or order d, where d is a nonnegative integer (Johansen, 1991).

If $\{x_t\}$ in (6) is cointegrated of order (1,1) with r cointegration vectors I know from Granger's Representation Theorem (GRT) that (i) rank $[A(1)] = r$, and (ii) $A(1) = \alpha\beta^T$ (see Engle and Granger (1987)). The matrices α and β are $n \times r$ and the columns of β are called the cointegration vectors. Under the assumption of cointegration it follows by GRT that an alternative form to (6) is:

$$A^*(L)\Delta x_t = \rho + \rho^* D_t - A(1)x_{t-1} + \varepsilon_t \tag{7}$$

Here, $\Delta \equiv 1 - L$ is the first difference operator, and $A(1) = I_n - \sum_{j=1}^{p} A_j$. The relationship between $A(L)$ and $A^*(L)$ is given by

$$A^*(L) = I_n - \sum_{i=1}^{p-1} A_i^* L^i \text{ , where } A_i^* = -\sum_{j=i+1}^{p} A_j \text{ for } i = 1,2,\dots,p-1$$

The representation in (7) is widely known as vector error correction (VEC) model. Cointegration implies that the r dimensional process $\{\alpha\beta^T x_t\}$ is jointly stationary. If I regard the cointegration vectors as describing a steady state or long run equilibrium for x, then the term $\alpha(\beta^T x_{t-1})$ represents the correction of the change in x_t due to last period's long run equilibrium error. Note that the major difference between equations (6) and (7) is that the latter representation is conditioned on cointegration while the former is merely consistent with unit roots.

2.2. The Restricted VAR Representation

Campbell and Schiller (1988) show that it is straightforward to rewrite the VEC representation as a restricted VAR system when $n = 2$ and $r = 1$. Theorem 1 in Warne (1993) shows that this result can be generalized. Let M be an $n \times n$ nonsingular matrix given by $[S_k^T \ \ \beta]^T$ where $S_k = \beta_\perp^T$ and $\beta_\perp^T \beta = 0$. Also let α^* be an $n \times n$ matrix equal to $[0 \ \ \alpha]$, while the $n \times n$ matrix polynomials $D(L)$ and $D_\perp(L)$ are

$$D(L) = \begin{bmatrix} I_k & 0 \\ 0 & (1-L)I_r \end{bmatrix}, \qquad D_\perp(L) = \begin{bmatrix} (1-L)I_k & 0 \\ 0 & I_r \end{bmatrix}$$

Next, let $z_t \equiv \beta^T x_t$, $\theta \equiv M\rho$, $\theta^* \equiv M\rho^*$ and $\eta_t \equiv M\varepsilon_t$. I can now derive a VAR representation for x_t which is conditioned on the cointegration vectors. I shall call this representation a restricted VAR (RVAR). Premultiplying both sides of (7) by M I get

$$MA^*(L)\Delta x_t = \theta + \theta^* D_t - M\alpha z_{t-1} + \eta_t$$

Define the n dimensional stationary vector y_t from $y_t \equiv D_\perp(L)Mx_t$. Noting that $(1-L)I_n = D(L)D_\perp(L)$ and $\alpha z_t = \alpha^* y_t$, I can express this system as:

$$B(L)y_t = \theta + \theta^* D_t + \eta_t \tag{8}$$

where $B(L) = M[A^*(L)M^{-1}D(L) + \alpha^* L]$

Note that $B(0) = I_n$, and that the matrix polynomial $B(L)$ is at most of order p. Since $\det[B(L)] = 0$ has all solutions outside the unit circle and $D_\perp(1)$ has rank r, it is clear that $A(1)$ has also rank r. The following version of GRT in Warne (1993) turns out to be very useful in the coming analysis of common trends:

Theorem: Suppose x_t is generated according to (6) with rank $[A(1)] = r < n$ and $\det[B(L)] = 0$ if and only if $|L| > 1$, then y_t, z_t and Δx_t is integrated of order zero. In addition

$$A(L) = M^{-1}B(L)D_\perp(L)M \tag{9}$$

and

$$C(L) = M^{-1}D(L)B(L)^{-1}M \tag{10}$$

where the Wold moving average representation of x_t is

$$\Delta x_t = \delta + C(L)[\varepsilon_t + \rho^* D_t] \text{ and } \delta = C(1)\mu \tag{11}$$

Note that the rank condition ensures that x_t is not integrated of order zero. The determinant condition, on the other hand, means that y_t in (8) has an invertible moving average representation and, accordingly y_t and thus z_t, is integrated of order zero. The rank condition implies that x_t is integrated of order one. Premultiplication by M^{-1} in (8) and using the definitions of y_t, θ and η_t gives us the expression in (9). Similarly, C(L) is obtained by premultiplying

$$y_t = B(1)^{-1}\theta + B(L)^{-1}\eta_t$$

by $M^{-1}D(L)$ and using the same definitions and the property that $(1-L)I_n = D(L)D_\perp(L)$.

In a sense, the theorem summarizes all I need to know about the reduced form mathematical properties of a vector time series which is cointegrated or order (1,1) with cointegrating rank r. The matrix polynomial $B(L)$ captures the general 'short-run' dynamics, whereas $(D_\perp(L), D(L))$ and M represent integration and cointegration, respectively. The important result here is that I have found a simple mathematical connection to the Wold Moving Average (MA) representation. Hence, the restricted VAR in (8) is very well suited for estimating a common trends model.

The unrestricted VAR in (6), the VEC representation in (7), and RVAR in (8) are all systems of linear stochastic difference equations. The mathematical solution to the three equations is given by (11). Thus, once the RVAR representation (8) has been estimated, the Wold MA representation (11), and hence, impulse responses can easily be obtained.

However, in order to construct y_t I need first to know the cointegration rank r and the cointegration vectors in the matrix β. These parameters can be estimated by Johansen's (1991) maximum-likelihood procedure. The starting point is then the representation (7). Under the hypothesis of cointegration $A(1)$ will have a reduced rank r. Johansen shows that it is possible to obtain estimates of the cointegration rank r and of the vector spaces spanned by the columns of α and β. In other words, I can estimate α and β up to a nonsingular transformation.

From the theorem it follows that the lag order of the restricted VAR in (8) is never greater than that of the unrestricted VAR in (6). In fact, unless all elements in the final r columns of the matrix A_p are zero the restricted VAR is also of order p. Given β, all other parameters can now be obtained through least-squares estimation of the RVAR representation. Alternatively, I can use Johansen estimates of $A^*(L)$ and α to calculate the $B(L)$ coefficients. Both procedures yield identical values for all parameters.

Let us consider $B(L) = I_n - \sum_{k=1}^{p} B_k L^k$. The theorem establishes that the matrix $C(1)$ is equal to $M^{-1}D(1)F(1)M$, where $F(1)$ is the inverse of $B(1)$. It then follows that if M, $\Omega \equiv M\,\Sigma M^T = E[\eta_t \eta_t^T]$ and $B(1)$ were known, so would Σ and $C(1)$ be.

2.3. The Common Trends Representation

Although the Wold representation in (11) is in MA form and thus suitable for analyses of impulse response functions, it has no clear economic interpretation since the disturbances ε are not structural. If I want to investigate the effects of a structural disturbance, e.g. one of the permanent shocks φ_t in the common trends model (3) and (4), I will have to put additional restrictions on my multivariate time series model. To see how the common trends parameters and the permanent shocks are identified, substitute recursively in (11) to obtain

$$x_t = x_0 + \delta t + C(L)(1 + L + L^2 + \ldots + L^t)[\varepsilon_t + \rho^* D_t] \tag{12}$$

which in turn may be written as

$$x_t = x_0 + C(1)\xi_t + C^*(L)[\varepsilon_t + \rho^* D_t] \tag{13}$$

The n-dimensional vector ξ_t is a random walk with time-varying drift according to

$$\xi_t = \rho + \rho^* D_t + \xi_{t-1} + \varepsilon_t$$

The relationship between the polynomials $C^*(L) = \sum_{i=1}^{\infty} C_i^* L^i$ and $C(L)$ is given by $C_i^* = -\sum_{j=i+1}^{\infty} C_j$.

The long run behaviour of x thus will be dominated by the nonstationary, random walk component $C(1)\xi_t$. But since $\beta^T x_t$ is stationary it must hold that $\beta^T C(1) = 0$, i.e. $C(1)$ must be of less than full rank. Specifically, from the definition of $C(L)$ under the equation (11) I know that $rank[C(1)] = rank[D(1)] = n - r$. This means that it is possible to rewrite (13) in terms of reduced number of independent trends, i.e. as a common trends model like (3) and (4). Stock and Watson (1988) derive the common trends model from the Wold representation (11) under the assumption of k common trends, and show that there are r cointegration vectors, $k = n - r$.

In order for $\beta^T x_t$ to be stationary the common trends coefficients Ψ in equation (3) must satisfy the following restrictions:

$$\beta^T \Psi = 0 \tag{14}$$

Given β, these cointegration restrictions provide $rk = (n-k)k$ equations which can be used to determine the nk parameters of Ψ. Additional restrictions may be derived from (3), (4) and (13), which imply that $\delta = C(1)\rho = \Psi\mu$ and $C(1)\varepsilon_t = \Psi\varphi_t$ for all t. It follows that

$$\Psi^T \Psi = C(1)\Sigma C(1)^T \tag{15}$$

Proceeding along the route suggested by KPSW (1991), I can write Ψ in the case of k common trends as

$$\Psi = \Psi_0 \pi \tag{16}$$

where Ψ_0 is an $n \times k$ matrix with parameters chosen so that $\beta^T \Psi_0 = 0$. A possible choice for Ψ_0 would be S_k. Using the relationship (15), and (16) I have that

$$\pi\pi^T = \left(\Psi_0^T \Psi_0\right)^{-1} \Psi_0 C(1)\Sigma C(1)^T \Psi_0 \left(\Psi_0^T \Psi_0\right)^{-1} \tag{17}$$

The right hand side of equation (17) is a $k \times k$ positive definite and symmetric matrix with $k(k+1)/2$ unique parameters. I cannot, however, solve for π uniquely without making some additional assumptions. For the above system of equations exactly $k(k+1)/2$ parameters can be uniquely determined, e.g. from a Choleski decomposition. It should be noted that although the

Choleski decomposition of π indicates a recursive structure for the influence of τ_t on x_t, the choice of Ψ_0 actually determines how the trends affect x_t. Thus, Ψ need not represent any recursiveness for the common trends model. To summarize this discussion, to identify the nk parameters of Ψ I first use the rk restrictions in (16). Hence, there remains to determine k^2 parameters. Second, I can solve the $k(k+1)/2$ independent equations in $\pi\pi^T$ if Ψ_0 is known. Accordingly, in addition to (16), $k(k-1)/2$ restrictions must be imposed on Ψ to achieve exact identification. These additional constraints should be motivated by economic theory since they cannot be tested.

2.4. Identification of Permanent and Transitory Innovations

My objective in this section is to be more specific about identification of all parameters in the common trends model. Two definitions and some new notation is introduced to minimize ambiguities.

Let Γ be any $n \times n$ nonsingular matrix such that $\Gamma\Sigma\Gamma^T$ is diagonal. The matrix $R(1) = C(1)\Gamma^{-1}$ is called the total impact matrix.

Definition 1. An $n \times n$ matrix Γ is said to identify a common trends model if (I) it is uniquely determined from the parameters of the model in equation (7), (ii) the total impact matrix is given by $R(1) = [\Psi \;\; 0]$.

Definition 2. An innovation ν_{it} is said to be permanent (transitory) if the i:th column of the total impact matrix is nonzero (zero).

From these two definitions it follows that if an $n \times n$ Γ identifies a common trends model, then the permanent innovations are those that are associated with the common trends. Let $n \times n$ nonsingular matrix Γ be chosen so that (i) the permanent innovations are equal to φ_t, (ii) the permanent and the transitory innovations, ψ_t, are independent, and (iii) the transitory innovations are mutually independent. I then have that

$$\Delta x_t = \delta + C(L)\varepsilon_t = \delta + R(L)\nu_t \qquad (18)$$

where $R(L) = C(L)\Gamma^{-1}$, $\nu_t = \Gamma\varepsilon_t$, and $E\{\nu_t \nu_t^T\} = I_n$. The component $R(L)\nu_t$ is called the impulse response function of Δx_t.

In order to derive a suitable matrix Γ, it may first be noted that

$$v_t = \begin{bmatrix} \varphi_t \\ \psi_t \end{bmatrix} = \begin{bmatrix} \Gamma_k \\ \Gamma_r \end{bmatrix} \varepsilon_t = \Gamma \varepsilon_t \tag{19}$$

where Γ_k and Γ_r are $k \times n$, and $r \times n$ matrices, respectively. It was already established that $\Psi \varphi_t = C(1)\varepsilon_t$ and that Ψ as well as $C(1)$ had rank equal to k. hence, it follows that the permanent innovations may be described by

$$\varphi_t = \left(\Psi^T \Psi\right)^{-1} \Psi^T C(1)\varepsilon_t \tag{20}$$

and, accordingly, the top $k \times n$ matrix Γ_k is $\left(\Psi^T \Psi\right)^{-1} \Psi^T C(1)$.

To find a matrix Γ_r which satisfies the conditions (ii) φ_t and ψ_t are independent, and (iii) the components of ψ_t are mutually independent, I first consider the condition (ii). Evaluating the covariance between the permanent and transitory innovations, I find that

$$E[\varphi_t \psi_t^T] = \left(\Psi^T \Psi\right)^{-1} \Psi^T C(1)\Sigma \Gamma_r^T \tag{21}$$

For this $k \times r$ matrix to be zero, it seems natural to let Γ_r include Σ^{-1}. That allows us to focus on the matrix $C(1)$, which is known to have reduced rank. From linear algebra it is well known that there exists exactly r linearly independent vectors which are orthogonal to the rows of $C(1)$. Letting $\Gamma_r = H_r \Sigma^{-1}$, I am therefore seeking an $r \times n$ matrix H_r such that $C(1)H_r^T = 0$. One possibility is to consider the space spanned by the columns of α. In fact, the following relationship may be established

$$\alpha = M^{-1} B(1) D_\perp(1) M \beta \left(\beta^T \beta\right)^{-1} \tag{22}$$

Premultiplying α by $C(1)$, I find that

$$C(1)\alpha = \left(M^{-1} D(1) F(1) M\right)\left(M^{-1} B(1) D_\perp(1) M \beta \left(\beta^T \beta\right)^{-1}\right) = 0$$

Let $H_r = Q^{-1}\zeta^T$, where Q is an $r \times r$ matrix, $\zeta = \alpha(U\alpha)^{-1}$, and U is an $r \times n$ matrix chosen so that $U\alpha$ is invertible (the specific use of U will become clearer in the attempt to identify the transitory disturbances). The covariance matrix for the transitory innovations is given by:

$$E[\psi_t \psi_t^T] = Q^{-1} \zeta^T \Sigma^{-1} \zeta \left(Q^T\right)^{-1} \tag{23}$$

A convenient normalization for the mutual independence of the transitory innovations is then to let $E[\psi_t \psi_t^T] = I_r$. Q can be chosen as the Cholesky decomposition of $\zeta^T \Sigma^{-1} \zeta$. The transitory innovations are now determined from

$$\psi_t = Q^{-1} \zeta^T \Sigma^{-1} \varepsilon_t \tag{24}$$

Accordingly, the matrix Γ_r is given by $\zeta^T \Sigma^{-1} \zeta$ so that the matrix Γ becomes

$$\Gamma = \begin{bmatrix} \left(\Psi^T \Psi\right)^{-1} \Psi^T C(1) \\ Q^{-1} \zeta^T \Sigma^{-1} \end{bmatrix} \tag{25}$$

It should be noted that the k linearly independent rows of Γ_k are linearly independent to the r linearly independent rows of Γ_r. These properties imply that Γ is of full rank.

The total impact matrix was identified as

$$R(1) = C(1)\Gamma^{-1} = [\Psi \ \ 0] \tag{26}$$

Let us partition the inverse of Γ into $\Gamma^{-1} = [\,\Gamma_k^+ \ \ \Gamma_r^+\,]$, where Γ_k^+ and Γ_r^+ are $n \times k$ and $n \times r$ matrices, respectively. Postmultiplying Γ in (25) by this expression for Γ^{-1}, I obtain:

$$\Gamma\Gamma^{-1} = \begin{bmatrix} \left(\Psi^T \Psi\right)^{-1} \Psi^T C(1)\Gamma_k^+ & \left(\Psi^T \Psi\right)^{-1} \Psi^T C(1)\Gamma_r^+ \\ Q^{-1} \zeta^T \Sigma^{-1} \Gamma_k^+ & Q^{-1} \zeta^T \Sigma^{-1} \Gamma_r^+ \end{bmatrix} = I_n$$

Letting $\Gamma_r^+ = \zeta \left(Q^T\right)^{-1}$ and $\Gamma_k^+ = \Sigma C(1)^T \Psi \left(\Psi^T \Psi\right)^{-1}$ I have found the inverse of Γ. Substituting for these relationships in (26), I have

$$R(1) = [R(1)_k \quad R(1)_r] = \left[C(1)\Sigma C(1)^T \Psi \left(\Psi^T \Psi\right)^{-1} \quad C(1)\zeta\left(Q^T\right)^{-1} \right]$$

It can easily be seen that $R(1)_k = C(1)\Sigma C(1)^T \Psi \left(\Psi^T \Psi\right)^{-1} = \Psi$ since $C(1)\Psi C(1)^T = \Psi\Psi^T$, while $R(1)_r = 0$ due to the fact that $C(1)\zeta = 0$. Using (19) it can be seen that $\Gamma\Sigma\Gamma^T = I_n$.

In order to derive the common trends model in (2) let us rewrite (13) as

$$x_t = x_0 + C(1)\xi_t + C^*(L)(\varepsilon_t + \rho^* D_t) \tag{27}$$

For this reduced from common stochastic trends representation I have that $\xi_t = \rho + \rho^* D_t + \xi_{t-1} + \varepsilon_t$ and $\delta = C(1)\rho$. In terms of equation (1) this means that

$$x_t^s = x_0 + C^*(L)(\varepsilon_t + \rho^* D_t)$$
$$x_t^p = C(1)\left[\xi_0 + (\rho + \rho^* D_t)t + \sum_{j=1}^{t} \varepsilon_j\right] \tag{28}$$

For the trend components in I have that

$$C(1)\xi_t = R(1)\Gamma\xi_t = R(1)\left[\Gamma\xi_0 + \Gamma(\rho + \rho^* D_t)t + \sum_{j=1}^{t} v_j\right] = \Psi\tau_t \tag{29}$$

In (29), $\mu = \Gamma_k \rho$, $\mu^* = \Gamma_k \rho^*$. Also from the stationary components I get $C^*(L)(\varepsilon_t + \rho^* D_t) = C^*(L)(\Gamma^{-1} v_t + \rho^* D_t)$ so that $\Phi(L) = C^*(L)\Gamma^{-1}$.

The matrix U can be used to give the transitory disturbances and economic interpretation. Suppose I want to identify the transitory innovations based on their contemporaneous relation to Δx (or to x). In that case with $R(0) = \Gamma^{-1}$ it follows that I should impose restrictions on Γ_r^+. To properly identify the transitory disturbances one needs $r(r-1)/2$ restrictions. Γ_r^+ is an $n \times r$ matrix, where rk restrictions come from $C(1)H_r^T = 0$ and $r(r+1)/2$ restrictions come from (23). Given α and Q the matrix U can always be chosen so that $r(r-1)/2$ in $\alpha(U\alpha)^{-1}(Q^T)^{-1}$ is zero. Now suppose Q is lower triangular (due to Cholesky decomposition) and that $n = 4$ and $r = 2$. Letting q_{ij}^+ and ζ_{ij} denote the (i,j):th elements of Q^{-1} and ζ respectively, I have that

$$\Gamma_r^+ = \begin{bmatrix} \zeta_{11} q_{11}^+ & \sum_{j=1}^{2} \zeta_{1j} q_{2j}^+ \\ \zeta_{21} q_{21}^+ & \sum_{j=1}^{2} \zeta_{2j} q_{2j}^+ \\ \zeta_{31} q_{31}^+ & \sum_{j=1}^{2} \zeta_{3j} q_{2j}^+ \\ \zeta_{41} q_{41}^+ & \sum_{j=1}^{2} \zeta_{4j} q_{2j}^+ \end{bmatrix}$$

To exactly identify the transitory innovations I need to consider one restriction on this matrix. A simple procedure is to let a particular element of ζ to be zero, say ζ_{11}. Then, the first transitory innovation has a zero contemporaneous effect on the first element of Δx. One can let the 2×4 matrix U be given by a zero-one matrix to satisfy the conditions for ζ.

2.5. A Structural Vector Error Correction Model of Real Exchange Rates

I specify a parsimonious four-dimensional VEC model with $x_t = [y_t \quad q_t \quad mp_t \quad i_t]^T$, comprising of real gross domestic product (y_t), real exchange rate (q_t) defined as foreign good price in terms of domestic goods, a measure of real balances (mp_t), where I use $M1$ as the money supply and GDP deflator to arrive at the real balances, and nominal interest rates (i_t) taken as the three month annualized rates for the treasury-bill. All variables are expressed in logs except the interest rates. Real gross domestic product (GDP), interest rates and real money balances are domestic values for the United States (US). Real exchange rate is defined as

$$q_t = \frac{ep^*}{p} \tag{30}$$

where e is the exchange rate, domestic currency per unit foreign currency; p^* and p are foreign and domestic consumer price indices, respectively.

One of the long run relations can be expressed as a standard money-demand relation that links real balances to real output and a measure of the opportunity cost of maintaining liquidity. In this VAR system, I expect to have at least one cointegrating vector for the semi-log money demand equation. [2]

$$mp_t = \theta_1 y_t - \theta_2 i_t + \varepsilon_t \quad : \theta_1, \theta_2 > 0 \tag{31}$$

The variables comprised in (30) and (31) are assumed to be governed by processes that exhibit stochastic trends, while the error term, ε_t can exhibit considerable persistence but is assumed to be stationary, so that (31) represents a long run or a cointegrating relation.

I also add a break based on prior knowledge of historical events. The break corresponds to the modification of U.S. monetary policy at the fourth quarter of 1979. [3]

In this four-dimensional system with one assumed cointegrating relation, there exist three stochastic tends and one transitory disturbance. [4] The transitory innovation encompasses all sorts of shocks to x_t with no permanent effects. Since there is only one transitory shock, there is no need to consider contemporaneous restrictions to identify the transitory shock, that is what is left out after all permanent shocks are accounted for. The permanent shocks, or the common stochastic trends, can be exactly identified after I impose three restrictions on the long run effects of the structural shocks on the variables in the VEC model as in Blanchard and Quah (1989, BQ for short). I name the three permanent shocks as supply, demand, and nominal shocks. The long run restrictions are imposed such that (i) nominal shocks have no long run effect on real output and real exchange rate, and (ii) demand shocks have no permanent effect on real output. With those restrictions x_t^P in (4) becomes

$$x_t^P = \begin{bmatrix} y_t^P \\ q_t^P \\ mp_t^P \\ i_t^P \end{bmatrix} = \Psi \tau_t = \begin{bmatrix} \Psi_{11} & 0 & 0 \\ \Psi_{21} & \Psi_{22} & 0 \\ \Psi_{31} & \Psi_{32} & \Psi_{33} \\ \Psi_{41} & \Psi_{42} & \Psi_{43} \end{bmatrix} \begin{bmatrix} \tau_{s,t} \\ \tau_{d,t} \\ \tau_{n,t} \end{bmatrix} \tag{32}$$

where Ψ_{ij} denote unrestricted elements, and $\tau_{s,t}, \tau_{d,t}, \tau_{n,t}$ are the supply, demand and nominal trends respectively. From Ψ it can be seen that real output is driven only by the productivity trend, real exchange rate by both productivity and demand trend, and the remaining real balances and interest rates by productivity, demand and nominal trends. In this way common trend parameters are exactly identified.

3. Estimation

I use quarterly observations for the G-7 countries (with U.S. as the home country) from 1973:2 to 1992:4. The data samples roughly correspond to the samples used by Clarida and Gali (1995) and I have expanded the set of countries by adding France and Italy[5]. It is assumed that y_t, q_t, mp_t, and i_t are unit root processes. Unit root tests are reported in Table 1. Both the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) fail to reject the unit root null hypothesis for real output, real money balances and interest rate for U.S. As for the real exchange rates only for U.K. ADF rejects unit root at 10% significance level.

I also performed unit root and stationarity tests for the differenced version of the series. The results indicate that each series appears to have at most one unit root. Thus, my results suggest that it is reasonable to treat all variables as having a single unit root, and thus leaving scope for stationarity through a possible linear combination.

Johansen brake trace tests for the number of cointegrating vectors in the system $\{y_t, q_t, mp_t, i_t\}$ are contained in Table 2.[6] The conventional trace critical values are also included to provide a comparison. However, they are not valid in the presence of breaks. The Akaike Information Criteria (AIC), Hannon-Quinn Test (HQ), and the Likelihood Ratio Test (LR) were applied to the four-dimensional unrestricted VAR in (6) to help determine the maximum lag length for the cointegration tests.[7,8] HQ consistently select fewer lags and provides a more parsimonious model. For all countries, I reject the null of no cointegration at both 5% and 10% significance when HQ lag selection is adopted. The AIC and LR tests consistently select the same lag length. However, some lag lengths border my VEC to more cointegrating vectors. The tests using the HQ criterion strongly conclude that Canada, United Kingdom and Italy have only one cointegrating vector regardless of the lag choice. At the 10% significance level France has three and Japan has two cointegrating vectors with AIC and LR lag selections. The AIC tests for Germany conclude at 90% significance that the rank of $A(1)$ is four which would mean all the variables are stationary which contradicts unit root tests. However, for Germany the assumption of only one cointegrating vector for a lag length of three may not be a good choice for the VEC model. The lag selection criteria suggest different values for the VEC model, in which case it is common practice to prefer Hannan-Quinn criterion. In addition HQ criterion results in stronger conclusions for the existence of only one cointegrating vector for most of the countries in my sample. Therefore, the rest of the analysis is carried out with a maximum lag length of two for Canada and one for all other G-7 member countries, under the assumption that the cointegrating rank is one.[9]

After identifying the existence of one cointegrating vector in line with my prior expectations, I test for stationarity and exclusion of the variables in the VEC model. Further supporting unit root tests, I reject stationarity of the variables for all countries. I expect to exclude the real exchange rate from the cointegrating vector as I expect to identify the money demand equation. However, only for Japan and United Kingdom can I exclude the real exchange rates from the cointegrating vector regardless of the significance level used. As for France and Italy real exchange rate can be excluded only at 5% significance, and Germany and Canada both include the real exchange rate in the cointegration space at 5%. For the common trend decomposition, I stick with my prior expectations and exclude the real exchange rate from the cointegration space.[10]

Table 4 outlines the estimated coefficients for all variables. Two separate cases are shown: (i) real exchange rate is part of the cointegration space, (ii) real exchange is excluded from the cointegrating vector. I have the correct signs of the coefficients in the money demand equation (31). An increase in real output increases real balances and higher interest rate lowers money demand. Real exchange rates move in the same direction with real output in the cointegrating vector when all variables are held constant. However, the coefficients for the real output are far from satisfying unitary income elasticity.

Table 5 performs the LR test of unitary income elasticity and strongly rejects it for all countries.

4. Identifying Structural Shocks from Impulse Responses

4.1 Point Estimates of Impulse Responses

Figures 1a, 1b, and 1c. illustrate the estimated impulse responses along with 90% confidence bands. [12] The lag lengths are the same as those used for the cointegration tests reported in Table 2. Kilian (1998) shows that constructing confidence bands for impulse responses in VARs using the asymptotically-correct normal approximation can be misleading in small samples. [11] Small sample bootstrap distributions are usually skewed (see Chapter II). Therefore I use the second-best Monte-Carlo integration approach to construct confidence bands around the impulse responses. Table 9 shows the results from applying Kolmogorov-Smirnov tests for normality based upon Monte-Carlo integration samples of a set of impulse responses for the U.S. – Japan real exchange rate. The bootstrap distribution of the impulse responses is skewed and normality is rejected. The histogram for the impulse response of real exchange rate for various shocks provides another visual test to judge for symmetry of the distribution.

I have put three restrictions on the stochastic trends for exact identification in the common trends model. I still need to compare the point estimates of the impulse responses with the qualitative expectations from common economic models in order to feel comfortable in naming them as permanent supply, demand, and nominal shocks. The standard exchange rate model (Dornbusch, 1976) predicts that an expansionary shock to U.S. monetary policy leads to depreciation in U.S. nominal and real exchange rates. Using a level VAR with short-run restrictions, Eichenbaum and Evans (1995) find evidence in favor of the prediction. Jang and Ogaki (2000) use a SVEC model of dollar / yen real exchange rate with long run restrictions and conclude that contractionary monetary shocks either using federal funds rates or nonborrowed reserve ratio lead to appreciation in the real exchange rate. CG (1995) use a trivariate VAR in differences and find that money supply shocks depreciate the real exchange rate. Rogers (1999) find that both the money multiplier and the monetary base shock cause real exchange rate depreciation in the short-run. I would also expect that a permanent positive nominal shock would permanently increase the nominal interest rate. The effect of a nominal shock on real output has been found significant, but temporary in models with slowly clearing market prices (CG, 1995).

In a similar four-variable VEC model with real output, nominal interest rates, real money balances and inflation with money demand as the cointegrating vector, Crowder, Hoffman and Rasche (1999, CHR for short) identify the permanent nominal shock as an inflation expectations shock. CHR provide external evidence that it tracks quite closely the Michigan inflation expectations survey. Their inflation expectations shock causes a temporary increase in real output due to a possible lag in interest rate increases leading to a lower real interest rate in the short-run and hence increasing output. Reduction of real balances prevails in both the short and the long run in response to an increase in the cost of maintaining money balances. A priori, I expect that my VEC model also lead to the same responses in the chosen variables. The permanent shock to inflation expectations can be precipitated by events that lead to upward revisions in inflationary expectations. I expect that that would lead to an increase in nominal interest rate and also a depreciation in the short-run to the real exchange rate.

Demand shocks, be it in the form of a change in government expenditures or a preference towards traded goods appreciate the real exchange rate (Rogers, 1999; CG, 1995). Such demand shocks can also cause a temporary increase in real output. So long as they lead to a change in the interest rates, I would expect a reduction in real balances both in the short and in the long run in line with money demand equation.

Productivity shocks are expected to increase real output as they change the capacity of the economy. As the marginal cost of capital is reduced, real money balances is expected to increase both through reduced interest rates and an increasing money supply from the monetary authority. The effect of productivity shocks on real exchange rates has often been a contentious issue for economists. CG model posits depreciation in the real exchange rate due to a supply shock. Their findings do not fully confirm their expectation and they note wrongly signed responses for part of their sample. MacDonald and Swagel (1998) also confirm the perverse sign of a supply shock on the real exchange rate. Chadha and Prasad (1997) find depreciation in the real exchange rates using real effective exchange rates and a larger sample than CG. Therefore, although there exists mixed empirical evidence for supply shock effects, my a priori expectation is that real exchange rate depreciates both in the short and in the long run.

Using common features of existing models in the literature my short-run and long run predictions of the directional effects of the permanent shocks on the variables in the VEC model would be: [13]

i) Short Run

	Supply	Demand	Nominal
Real GDP	>0	>0	>0
Real Exchange	>0	<0	>0
Real Money Balances	>0	<0	<0
Nominal Interest Rate	<0	>0	>0

ii) Long Run

	Supply	Demand	Nominal
Real GDP	>0	0	0
Real Exchange	>0	<0	0
Real Money Balances	>0	<0	<0
Nominal Interest Rate	<0	>0	>0

Consider first the point estimates of the impulse responses. I want to examine whether the signs of the short and long run responses of U.S. real output, the U.S. real exchange rate, U.S. real M1 money balances, and the U.S. treasury-bill interest rate to positive one standard deviation supply, demand, and nominal shocks are consistent with my expectations.

The impulse responses for Germany, France, Japan and Italy meet all my expectations. [14] However, there are discrepancies between the estimated responses and my predictions. Real exchange rate appreciates for United Kingdom as in CG. All other responses have correct signs for United Kingdom. The common trend decomposition does not fare well with Canada: Supply shock appreciates the real exchange rate in the long run; demand shock reduces output, appreciates the real exchange rate and reduces the nominal interest rate in the short-run; the nominal shock appreciates the real exchange rate in the short-run. Those responses do not provide a proper shock identification for Canada. Overall, I have quite satisfactory performance from the impulse responses and I am justified in interpreting the shocks as supply, demand, and inflation expectations shocks. I do not try to identify the transitory shock as there is only one and it is hard to attach an economic meaning for it is a composite of all types of transitory shocks.

4.2. Confidence Intervals

Point estimates only tell part of the story and sampling error should be taken into account through the construction of confidence intervals. The inflation expectations shock significantly increases real output in the short-run, decreases real balances and increases nominal interest rates both in the short and in the long run for all countries. Real exchange rates do not have significant responses to nominal shocks except for Japan in the short-run. Demand shock responses are significant for real output for all G-7 members, with Canada having the opposite sign. Real exchange rate also significantly appreciates. Only for United Kingdom do the nominal interest rate and real balances not have significant responses. The productivity shock significantly increases real output after the first two quarters. Real money balances exhibit a significant increase and the nominal interest rate a significant decrease for all G-7 countries. As I had expected I have a slightly mixed response from real exchange rates to supply shocks. Only for Japan do I have significant responses, whereas the other countries show insignificance of point estimates at all horizons.

4.3. Robustness: estimating with different lag lengths

There are different ways one can handle the choice of lag lengths in a VAR framework. Among the alternative procedures commonly used are Schwartz Bayesian Criteria (SBC), the Akaike Information Criteria (AIC), Hannan-Quinn Criteria (HQ), and the Likelihood Ratio Test (LR). In this section, I consider how robust the structural VEC model results are to different lag lengths.

The results presented above were based on HQ criteria. In five cases HQ selected lag length one and only for Canada it selected lag length two (Table 3). AIC and LR picked the same but higher lag lengths than HQ for all countries. Figure 2 illustrates how the results for Germany, Japan and France differ according to whether the lag lengths are chosen via HQ or LR, and AIC.[15] I observed that the response of real exchange rate to a supply shock was the most sensitive impulse response. French and Japanese real exchange rate appreciates about five quarters after the supply shock and then depreciates with a lag length of two, while the impulse response for lag length one quickly depreciates after the shock. France and Japan have perverse signs for the supply shock only in the short-run. However, the responses from Germany are quite different. It displays the perverse sign effect both in the short and in the long run. Real exchange rate experiences a permanent depreciation for a lag choice of three, while with HQ criterion and a lag equal to one, it depreciates. One possible explanation might be that unaccounted cointegration effects might be what is behind the changing sign of the responses. From Table 2, I see that only for Germany do I have evidence that there exists more than two cointegrating vectors. As for Japan and France the evidence is that there may exist two cointegrating vectors at the most. Based on cointegration tests, Germany seems to be the only country where I may have a serious misspecification problem for the common trends decomposition with one cointegrating vector.[16]

4.4. Variance Decompositions

Table 6.a provides decompositions of the k-step ahead forecast error variance in the forecasting the logged level of the real exchange into the proportion attributable to each other structural innovations. Under the assumption that the real exchange rate is a unit root process, the unconditional variance of the real exchange rate, which is the limit of the k-step forecast error variance as k goes to infinity, is not finite. However, the unconditional variance of the stationary first-differenced exchange rate is finite. Its decomposition can be inferred from Table 6.b, which provides decompositions of the conditional variance of the differenced log of the real exchange rate into the proportion attributable to each of the structural innovations. Table 6 also includes the 90-percent confidence intervals for each estimated proportion. Warne (1993) shows that they are normally distributed. However, in small samples the distribution is fairly skewed. Figure 4 illustrates for the Japanese case, that the bootstrap distributions of the variance decompositions. The distributions for the supply, nominal and the temporary shock are skewed to the left. Similar skewness was observed for most variance decompositions. As a result, I chose to report 90-percent confidence intervals derived from the empirical bootstrap using Monte-Carlo integration.

First, consider the conditional variances of (logged level of the) real exchange rate reported in Table 6.a. For all six countries, demand shocks dominate in the short-run and in the long run. For Germany, France and Italy demand shocks account for over 90% of the forecast error variance for all horizons. Japan, Canada, and UK accounts for at least close to 80% until 12 quarters. Demand shocks explain more than 90% at horizon 24 for Canada and UK, but only 78% for Japan. Supply shocks dominate over nominal shocks after 4 quarters. In Japan supply shocks account for 20% of the forecast error variance after 24 quarters. Nominal shocks exert their biggest influence in the first quarter and quickly lose their effect in variance decompositions. A similar pattern is observed for the temporary shocks. After 24 quarters demand shocks dominate, followed by supply and nominal shocks.

Next consider Table 6b., which presents the forecast-error variance decompositions for the differenced logs of the real exchange rate. The proportion of the k-step forecast-error variance in the differenced real exchange rate attributable to a particular type of shock will converge as k increases to the proportion of the unconditional variance attributable to that shock. In Table 6b I observe that convergence occurs within the first 12 quarters. I also observe that demand shocks account for more than 70% at all horizons. After conversion occurs, they account from 74% to 93% of the forecast error variance. Supply shocks seem to be the second dominant influence on real exchange rate changes. Supply shocks dominate nominal shocks after 4 quarters and at 24 quarter horizon their effect ranges from 3% to 11%. Nominal shocks tend to explain similar amounts at all horizons. Temporary shocks exhibit significant effect on forecast error variances. For Japan they explain 10%, for Canada 16% and for UK 5% after 24 quarters.

I also compare my variance decompositions with findings from Chapter two in Table 7. Chapter two follows CG approach with three variables, real output, real exchange rate and price level differences and carry out a structural VAR approach in differences. Their decomposition has the same identification constraints on the effects of shocks. Their variance decomposition has the same supply, demand and nominal shocks, shocks having no permanent effect on the output and real exchange rate. A similar decomposition can be shown for my model after I aggregate permanent nominal and temporary shocks as neither of them have any permanent effect on real exchange rate and real output. Demand and supply shocks are also aggregated as a different column as permanent shocks. UK, Germany, France and Italy have surprisingly close values for the forecast error variance decompositions of differenced real exchange rates when it comes to identifying the forecast error variance explained by real and transitory shocks after 24 quarters. For example, permanent shocks explain about 96.8 % in my model whereas CG model in Chapter two explains % 96.9. The relative effects are different for Canada and Japan.

4.5. An Examination of the Innovations that Compose the Permanent Trend

I can investigate whether the common trends identified by my decomposition may be ascribed to trends in government expenditures and inflation expectations for the case of France. The trend for the demand component is the accumulated demand shocks identified in the VEC model and similar shocks are expected to affect the ratio of federal expenditures to output. I called my identified nominal shocks as the inflation expectations shock and I may compare the nominal trend with Michigan Survey Data. Inflationary expectations are compiled by the Survey

Research Center at the University of Michigan. Participants are polled for their estimates of the future course of inflation over the subsequent twelve-month period. A comparison of the ratio of federal expenditures in U.S. to identified trend for the demand component appears in Figure 3, along with the comparison of Michigan Survey and the identified nominal trend.[17] The first figure reveals that the trend rate of growth in inflation expectations is clearly mimicked by the nominal trend. The decreasing trend is evident after the New Operating Procedures of the FED in 1979 and after the resumption of interest rate targeting after 1981. The correlation coefficient is 0.66. The second figure about the trend for the demand component shows similar trend growth in the early Reagan in years (1980-1984) when federal expenditures were on the rise, and then a decline toward the end of Reagan's second term in office. The demand component continues its declining trend while the ratio of federal expenditures to output is stabilized after 1988. The correlation coefficient between the two series is 0.51. the remaining correlation coefficients for Canada, Japan, UK, Italy and Germany are in Table 8. The correlation coefficients for the permanent demand component ranges from 0.35 to 0.52. The correlation coefficients for the nominal trend range from 0.07 to 0.66. Canada shows the lowest amount correlation. This is not surprising since I was not able to correctly identify the shocks in the Canadian case.

5. Conclusion

The fundamental innovations I identified in my simple cointegrated system leave imprints that coincide with predictions of most contemporary macroeconomic and international finance theories about real exchange rates. Rather than just asserting long run restrictions on the effects of shocks for the chosen VAR variables, I first establish cointegration rank, and thereby enumerate distinct, independent, permanent innovations that underlie the system. Identification is guided by knowledge of the long run money demand relationship that ties the variables in the system together. Two of the permanent shocks have the characteristics of a supply, and an aggregate demand innovation, whereas the permanent nominal shock bears features of a shock to inflation expectations.

Shock identification for real exchange rates in a structural VAR model has generally been carried out using differenced variables with long run restrictions. Such approaches are incomplete for there exists evidence of cointegrating relationships between the macroeconomic variables used in the VAR estimations. Even stylized paper by CG (1995) is burdened with cointegrating relationships for half of their sample. Chapter two also pointed to possible misspecifications in structural VAR systems where cointegration effects are ignored. The point estimates of impulse responses may even change signs leading to serious difficulties in identification. For a more general approach Alexius (2000), Karstensen and Hansen (1997), and Ogaki (2001) attempt at structural shock identification using common trends approach for real exchange rates. My study differs from them by (i) incorporating all G-7 countries in the estimations, (ii) using an economic behavioral relationship, money demand function as the cointegrating vector, (iii) using only the floating exchange rate period for comparison with other structural VAR studies, (iv) illustrating the adverse effects of unaccounted cointegration relationships on impulse responses.

Common trends approach for the structural decomposition of real exchange rates corrects for the wrong sign of impulse responses in the presence of cointegration (Chapter two). For most of the countries in the sample, the perverse sign effect of the supply shock is eliminated. Real exchange rate appreciates only for UK in response to a supply shock, which is also confirmed by other studies (CG, 1995; MacDonald and Swagel, 1998). I observe more significant effects of structural shocks on the variables in the common trends approach. That further illustrates the benefits of common trends decompositions. Nominal and supply shocks turned out to be insignificant for real exchange rates for most countries and significance of shocks on real exchange rates still remains a problem. One of the most serious effects of cointegration in the structural decompositions was examined on real exchange rate responses to supply shocks. Model mispecifications can result in diametrically opposite conclusions as impulse responses change sign with different lag lengths that bring along additional co-movement (cointegration) of the variables.

The variance decompositions confirm the previous results in the literature, demand shocks being the dominant factor in real exchange rate fluctuations. One other important feature of structural decompositions I identify is that it is the type of long run restrictions rather than the variables used that change the relative importance of permanent versus nominal shocks on the real exchange rates. Although my common trends decomposition uses different variables, the forecast error variance that can be explained by permanent innovations for real exchange rate differences are very similar for most of my sample which is the same for Chapter two even differing only by 0.1% points. Sarte and Daniel (1997) point out that identification in structural VARs can be very sensitive to identifying restrictions. My restrictions are the same with Chapter two, which is why I have similar results in variance decompositions.

The structural VEC model of real exchange rates in the floating exchange rate period pursued in this paper provides a more general estimation approach and can be regarded as an improvement on the existing structural decomposition techniques.

Notes

1. Actually, Clarida and Gali (1995) find cointegration for two of the four countries at 10% significance level, but they ignore this in the subsequent empirical analysis. In Rogers (1999), the null hypothesis of cointegration would not be rejected if the 90% critical values of the Johansen (1988) tests were used instead of 95% critical values.

2. Unitary income elasticity is a testable proposition that I examine in my empirical analysis. Evidence of cointegration among these variables have been examined in various papers. Hoffman and Rasche (1991b), Hoffman et al. (1995), and Stock and Watson (1993) present evidence on the stationarity of money demand.

3. The break at 79:4 capture shifts in the deterministic drift of the variables triggered by the New Operating Procedures experiment that began in October 1979. Crowder, Hoffman and Rasche (1999) pursue a similar approach. I do not include the break for the resumption of interest rate targeting, as it does not significantly alter the implications of common trends approach, but yet reduce the performance of the decomposition mainly in the short-run. The break variable is 0 prior and 1.0 as of 1979:4. I found the break dummies to be significant for real money balances for all cases.

4. Using the notation of sections 2.1 through 2.3, $n = 4, k = 3$, $r = 1$ and $\beta^T = [-\theta_1 \quad 0 \quad 1 \quad \theta_2]$.

5. Clarida and Gali (1995) use quarterly data 1976:3 – 1992:4 for Japan, 1974:3 – 1992:4 for Canada, Britain and Germany. Their data sources are as follows: CPI data are from International Financial Statistics (IFS) data tapes, real GDP data are from OECD Main Economic Indicators and exchange rate data are from Federal Reserve Bank of New York. My data for all G-7 countries are from IFS December 1993. I did not extend the data at this point to current observations as the objective of this paper is to see if error correction can rectify the identification and lag selection problems exposed in Chapter two.

6. I do not report the usual λ_{max} test statistics as their distribution in the presence of breaks has not yet been determined.

7. The analysis for unit root and cointegration has been done using RATS and also MALCOLM 2.4 (Mosconi, 1998) which is also a menu-driven RATS package implemented in a user-friendly environment.

8. Using the notation of sections 2.1 through 2.3, $p = 2$, for Canada, and $p = 1$ for all the remaining countries. For the maximum lag selection, I checked the residuals in the VAR estimation in levels and increased lag length until I was satisfied that the residual behaviour was consistent with white noise errors.

9. Using different lag lengths do not cause significant changes in the common trends decomposition. Higher lag lengths lower the performance of the decomposition for the short-run, with the exception of Germany that results in a sign change in the impulse responses with a lag length of three both in the short-run and the long run. Those issues will be re-examined in the section about the robustness of the impulse responses to lag selection in the common trends model.

10. Including real exchange rate alters the impulse responses slightly only in the short-run, and do not affect the main conclusions of the paper.

11. Kilian recommends a bootstrap-after-bootstrap method to get more reliable confidence bands for impulse responses. The second-best approach in Kilian (1998) is Monte-Carlo integration and that is what I pursue in this paper.

12. The common trend decomposition was carried out using part of the procedures in RATS written by Anders Warne. The procedures are available in the owner's website. In the impulse responses, rgdp is real GDP, rexch is the real exchange rate, rmb is the real money balances, and int is the nominal interest rate.

13. The loading matrix Ψ is determined up to a sign of its coefficients after proper choice of Ψ_0 due to the Cholesky decomposition to find π. I resolve these ambiguities by trying to match the predictions for certain long run effects of the structural shocks with the estimated long run responses. These long run selection criteria are: supply shocks have a positive effect on real GDP, demand shocks appreciate the real exchange rate, and nominal shocks increase the nominal interest rate.

14. It is not very clear what economists exactly mean by short-run. If short-run is defined as the total effect in the first four quarters after the shock, then I have the correct signs for the short-run. The real exchange rate for Germany and Japan depreciates the first quarter after the shock and then depreciates quickly as expected. The real output for Japan also dips in the first quarter and then takes off.

15. We do not present figures for Canada, and UK as the responses displayed very minor changes with different lag lengths. I believe the non-robustness of impulse responses are closely related to cointegration effects and therefore chose only Germany, Japan and France where Table 2 hints at more than one cointegration vectors when significance level is set at 10%.

16. Using lag length three, the impulse responses of the other variables due to a supply shock for Germany also has perverse signs in the VEC model. Real balances decrease permanently after a supply shock, and nominal interest rates are reduced in the short-run for three-quarters after the shock.

17. The graphs have two separate scales. The scale on the left is for the identified trend components, and the scale on the right is for ratio of federal expenditures to output and for the Michigan Survey. The reason I have two scales is that I do not have the variables inflation and federal expenditures to GDP ratio in my structural VEC model. The identified trends need to satisfy some simple accounting being that their sum equal to the permanent components of the variables in the system. Should I have inflation and federal expenditure ratio, then the identified trend components would be anchored better to the

scales of the variables to which are they are most related. I am mainly interested in the trend growth and hence the different scales are not my main concern.

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TABLE 1

UNIT ROOT TEST RESULTS

Variable		Country					
		Canada	Japan	U.K.	Germany	France	Italy
ADF Tests:							
q	τ_μ	-2.051	-1.549	-2.874*	-1.793	-2.116	-1.828
United States							
y	τ_τ	-2.433					
	τ_μ	-0.659					
mp	τ_τ	-1.953					
	τ_μ	0.015					
i	τ_μ	-2.190					
PP Tests:							
q	τ_μ	-1.62	-1.44	-1.52	-1.58	-1.69	-1.35
United States							
y	τ_t	-2.32					
	τ_μ	-0.36					
mp	τ_t	-1.80					
	τ_μ	0.57					
i	τ_μ	-1.56					

Notes: y is logged real GDP, q is logged real exchange rate, mp is logged real money balances, and i is the Treasury bill rate on an annual basis. ADF is the Augmented Dickey-Fuller test statistics. The number of lags included in the ADF regressions were selected by the AIC. PP is the Phillips-Perron test statistics, constructed using the first four autocovariances. Critical values from the Dickey-Fuller distributions with sample size 100 were used for all countries.

* denotes significant at the 10% level and ** denotes significant at the 5% level.

TABLE 2

Tests for Cointegration and Identifying the Cointegration Space in the VAR System

$x_t = [\, y_t \quad q_t \quad mp_t \quad i_t \,]$

Countries	Lag Criteria	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
CANADA	HQ (2)	56.69**	19.04	5.96	0.01
	AIC (3)	41.44	17.18	6.16	0.01
	LR (3)	41.44	17.18	6.16	0.01
JAPAN	HQ (1)	83.30**	19.36	3.87	2.77
	AIC (2)	57.24**	25.85*	7.72	0.32
	LR (2)	57.24**	25.85*	7.72	0.32
UNITED KINGDOM	HQ (1)	71.71**	14.37	5.39	0.67
	AIC (2)	54.73**	19.69	6.81	0.95
	LR (2)	54.73**	19.69	6.81	0.95
GERMANY	HQ (1)	81.07**	17.89	6.09	1.09
	AIC (3)	57.30**	25.47*	13.06*	4.86*
	LR (3)	57.30**	25.47*	13.06*	4.86*
FRANCE	HQ (1)	77.79**	17.10	5.49	0.81
	AIC (2)	57.48**	23.82*	9.29*	1.77
	LR (2)	57.48**	23.82*	9.29*	1.77
ITALY	HQ (1)	77.62**	15.66	2.94	0.73
	AIC (2)	56.04**	22.94	7.33	0.53
	LR (2)	56.04**	22.94	7.33	0.53

"Break" Trace Critical Values

Test	$r = 0$	$r \leq 1$	$r \leq 2$	$r \leq 3$
95%	45.84	26.23	9.51	3.84
90%	42.53	23.75	7.77	2.71
Conventional Trace Critical Values				
95%	47.20	29.70	15.40	3.80
90%	44.00	26.80	13.30	2.70

Notes: Trace critical values are based on results of simulations designed to account for the possibility of breaks in the deterministic drifts of the data corresponding to the intervention dummy at 79:4. The simulations were performed on DisCo and the experiments were designed as prescribed by Johansen and Nielsen (1993). Conventional critical values are taken from Osterweld-Lenum (1992).

HQ is the Hannan-Quinn criterion, AIC is the Akaike Information Criterion, and LR is the Likelihood Ratio Test. Lag is the lag length in VAR with levels series. Due to the quarterly nature of the data, the model was pared down starting from a maximum lag length of five. The values in parentheses are the chosen lag lengths. Significance of coefficients are determined using "Break" Trace critical values.

TABLE 3

Tests of Stationarity and Exclusion p-values of Tests against indicated null

		y_t	q_t	mp_t	i_t
<u>Countries</u>					
CANADA					
	H_S	0.000	0.000	0.000	0.000
	H_E	0.040	0.035	0.000	0.000
JAPAN					
	H_S	0.000	0.000	0.000	0.000
	H_E	0.112	0.741	0.000	0.000
UNITED KINGDOM					
	H_S	0.000	0.000	0.000	0.000
	H_E	0.013	0.186	0.000	0.000
GERMANY					
	H_S	0.000	0.000	0.000	0.000
	H_E	0.023	0.026	0.000	0.000
FRANCE					
	H_S	0.000	0.000	0.000	0.000
	H_E	0.012	0.075	0.000	0.000
ITALY					
	H_S	0.000	0.000	0.000	0.000
	H_E	0.009	0.050	0.000	0.000

Notes: The hypothesis H_S refers to the null that the variable is integrated of order zero. H_E refers to the null that the cointegration space has zero coefficients for the variable (i.e. it is excluded from the cointegration space). H_S is distributed as χ^2 with three degrees of freedom, and H_E as χ^2 with one degree of freedom.

TABLE 4
Cointegrating vector Coefficients

	y_t	q_t	mp_t	i_t
Countries				
CANADA	-0.401	0.417	1	0.050
	-0.352	0.000	1	0.044
JAPAN	-0.368	0.032	1	0.051
	-0.317	0.000	1	0.051
UNITED KINGDOM	-0.474	0.096	1	0.045
	-0.414	0.000	1	0.046
GERMANY	-0.485	0.183	1	0.053
	-0.402	0.000	1	0.049
FRANCE	-0.497	0.129	1	0.048
	-0.426	0.000	1	0.047
ITALY	-0.576	0.198	1	0.054
	-0.423	0.000	1	0.046

TABLE 5
Likelihood Ratio Test of Unitary Income Elasticity

	χ^2 $(df = 2)$	p-value
Countries		
CANADA	21.179	0.000
JAPAN	24.146	0.000
UNITED KINDOM	17.763	0.000
GERMANY	21.323	0.000
FRANCE	19.102	0.000
ITALY	21.482	0.000

Notes: The cointegrating vector coefficients table includes coefficients for the case where all variables are included in the cointegration space and also for the case when only money demand equation is assumed to define the comovements of the variables. The Likelihood Ratio tests exclude real exchange rate from the cointegration space and assumes unit income elasticity for the semi-log money demand equation.

The cointegrating coefficients are normalized on real money balances.

TABLE 6
DECOMPOSITIONS OF K-STEP-AHEAD FORECAST ERROR VARIANCE IN REAL EXCHANGE RATES ATTRIBUTABLE TO EACH TYPE OF SHOCK WITH 90-PERCENT MONTE-CARLO CONFIDENCE INTERVALS

Part a: Real Exchange Rate Levels

U.S. – Japan

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock	Temporary shock
1	0.014 (0.000,0.138)	0.829 (0.577,0.955)	0.054 (0.005,0.147)	0.102 (0.003,0.281)
4	0.056 (0.011,0.271)	0.879 (0.636,0.969)	0.027 (0.003,0.081)	0.060 (0.007,0.148)
12	0.162 (0.013,0.544)	0.814 (0.422,0.976)	0.007 (0.001,0.020)	0.016 (0.002,0.034)
24	0.206 (0.011,0.648)	0.784 (0.337,0.984)	0.003 (0.000,0.008)	0.007 (0.001,0.13)

U.S. – Germany

Forecast Horizon:	Supply Shock	Demand Shock	Nominal shock	Temporary Shock
1	0.003 (0.000,0.154)	0.907 (0.702,0.981)	0.024 (0.000,0.072)	0.065 (0.000,0.182)
4	0.031 (0.004,0.175)	0.936 (0.759,0.988)	0.008 (0.000,0.034)	0.025 (0.001,0.080)
12	0.090 (0.004,0.396)	0.900 (0.577,0.992)	0.002 (0.000,0.009)	0.007 (0.000,0.021)
24	0.111 (0.003,0.527)	0.884 (0.466,0.995)	0.001 (0.000,0.003)	0.003 (0.000,0.008)

U.S. – Canada

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock	Temporary shock
1	0.042 (0.001,0.162)	0.793 (0.554,0.955)	0.020 (0.000,0.087)	0.144 (0.012,0.313)
4	0.017 (0.004,0.146)	0.928 (0.757,0.978)	0.013 (0.000,0.060)	0.041 (0.004,0.106)
12	0.012 (0.004,0.253)	0.971 (0.706,0.988)	0.004 (0.000,0.018)	0.012 (0.002,0.031)
24	0.014 (0.003,0.346)	0.977 (0.636,0.992)	0.002 (0.000,0.007)	0.006 (0.000,0.013)

U.S. – France

Forecast Horizon:	Supply Shock	Demand Shock	Nominal shock	Temporary Shock
1	0.001 (0.000,0.085)	0.974 (0.798,0.990)	0.021 (0.000,0.074)	0.004 (0.000,0.113)
4	0.043 (0.002,0.212)	0.947 (0.730,0.991)	0.005 (0.000,0.032)	0.004 (0.000,0.055)
12	0.076 (0.003,0.386)	0.920 (0.598,0.993)	0.002 (0.000,0.009)	0.001 (0.000,0.018)
24	0.088 (0.003,0.490)	0.910 (0.497,0.995)	0.001 (0.000,0.004)	0.001 (0.000,0.007)

U.S. – UK

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock	Temporary shock
1	0.102 (0.008,0.263)	0.857 (0.624,0.961)	0.004 (0.000,0.045)	0.037 (0.000,0.171)
4	0.076 (0.008,0.283)	0.910 (0.678,0.977)	0.003 (0.000,0.032)	0.011 (0.001,0.060)
12	0.042 (0.008,0.350)	0.953 (0.645,0.986)	0.001 (0.000,0.012)	0.003 (0.000,0.020)
24	0.031 (0.006,0.427)	0.966 (0.569,0.991)	0.000 (0.000,0.005)	0.002 (0.000,0.009)

U.S. – Italy

Forecast Horizon:	Supply Shock	Demand Shock	Nominal shock	Temporary Shock
1	0.000 (0.000,0.070)	0.981 (0.820,0.988	0.019 (0.000,0.074)	0.000 (0.000,0.101)
4	0.032 (0.002,0.189)	0.959 (0.769,0.990)	0.005 (0.000,0.034)	0.003 (0.001,0.049)
12	0.061 (0.003,0.355)	0.936 (0.625,0.994)	0.002 (0.000,0.010)	0.001 (0.000,0.016)
24	0.071 (0.003,0.436)	0.928 (0.554,0.995)	0.001 (0.000,0.004)	0.000 (0.000,0.007)

Table 6, Part b: Real Exchange Rate Differences

U.S. – Japan

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock	Temporary shock
1	0.014 (0.000,0.138)	0.829 (0.577,0.955)	0.054 (0.055,0.147)	0.102 (0.003,0.281)
4	0.085 (0.033,0.229)	0.769 (0.519,0.914)	0.049 (0.007,0.122)	0.096 (0.016,0.235)
12	0.111 (0.044,0.342)	0.738 (0.447,0.899)	0.050 (0.007,0.116)	0.099 (0.017,0.226)
24	0.111 (0.044,0.349)	0.738 (0.437,0.899)	0.050 (0.007,0.116)	0.099 (0.017,0.226)

U.S. – Germany

Forecast Horizon:	Supply Shock	Demand Shock	Nominal shock	Temporary Shock
1	0.003 (0.000,0.154)	0.907 (0.702,0.981)	0.024 (0.000,0.072)	0.065 (0.001,0.182)
4	0.051 (0.014,0.188)	0.859 (0.654,0.964)	0.024 (0.001,0.065)	0.064 (0.005,0.166)
12	0.057 (0.016,0.237)	0.852 (0.605,0.961)	0.024 (0.001,0.063)	0.066 (0.005,0.165)
24	0.057 (0.016,0.237)	0.852 (0.605,0.961)	0.024 (0.001,0.063)	0.066 (0.005,0.165)

U.S. – Canada

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock	Temporary shock
1	0.042 (0.001,0.162)	0.793 (0.554,0.955)	0.020 (0.000,0.087)	0.144 (0.012,0.313)
4	0.054 (0.012,0.175)	0.754 (0.523,0.922)	0.022 (0.001,0.087)	0.169 (0.022,0.337)
12	0.058 (0.015,0.195)	0.749 (0.511,0.917)	0.022 (0.002,0.085)	0.169 (0.023,0.331)
24	0.058 (0.015,0.201)	0.749 (0.511,0.917)	0.022 (0.002,0.085)	0.169 (0.023,0.331)

U.S. – France

Forecast Horizon:	Supply Shock	Demand Shock	Nominal shock	Temporary Shock
1	0.001 (0.000,0.085)	0.974 (0.797,0.990)	0.021 (0.000,0.074)	0.004 (0.000,0.113)
4	0.036 (0.008,0.158)	0.931 (0.712,0.976)	0.025 (0.001,0.078)	0.008 (0.002,0.123)
12	0.037 (0.010,0.183)	0.930 (0.683,0.974)	0.025 (0.001,0.076)	0.008 (0.003,0.119)
24	0.037 (0.010,0.188)	0.930 (0.680,0.974)	0.025 (0.001,0.076)	0.008 (0.003,0.119)

U.S. – UK

Forecast Horizon:	Supply Shock	Demand Shock	Nominal Shock	Temporary shock
1	0.102 (0.008,0.263)	0.857 (0.624,0.961)	0.003 (0.000,0.045)	0.037 (0.000,0.171)
4	0.101 (0.021,0.277)	0.850 (0.606,0.939)	0.004 (0.001,0.049)	0.044 (0.003,0.194)
12	0.102 (0.025,0.282)	0.848 (0.597,0.935)	0.005 (0.000,0.051)	0.045 (0.003,0.191)
24	0.102 (0.026,0.283)	0.848 (0.597,0.935)	0.005 (0.000,0.051)	0.045 (0.003,0.191)

U.S. – Italy

Forecast Horizon:	Supply Shock	Demand Shock	Nominal shock	Temporary Shock
1	0.000 (0.000,0.071)	0.981 (0.820,0.988)	0.019 (0.000,0.074)	0.000 (0.000,0.095)
4	0.030 (0.007,0.141)	0.938 (0.735,0.972)	0.025 (0.002,0.080)	0.006 (0.003,0.121)
12	0.031 (0.009,0.162)	0.937 (0.699,0.969)	0.025 (0.002,0.80)	0.007 (0.004,0.122)
24	0.031 (0.009,0.162	0.937 (0.693,0.969)	0.025 (0.002,0.80)	0.007 (0.004,0.122)

TABLE 7

Comparison of Forecast Error Variance Decompositions after 24 quarters

Countries	Model	Permanent demand + supply	Permanent nominal + temporary
CANADA	CG	0.916	0.084
	CT	0.807	0.193
JAPAN	CG	0.957	0.043
	CT	0.849	0.151
UNITED KINDOM	CG	0.948	0.052
	CT	0.950	0.050
GERMANY	CG	0.913	0.087
	CT	0.909	0.091
FRANCE	CG	0.970	0.030
	CT	0.967	0.033
ITALY	CG	0.969	0.031
	CT	0.968	0.032

Notes: The values in the table are the forecast error variance of changes in real exchange rates that can be explained by the relevant shocks using CG (Clarida and Gali) model, and CT (Common Trends) model after 24 quarters. CG model values are from page 54 in Chapter II. CG has 3 variables: relative real GDP, real exchange rate, and relative price levels. CT model has 4 variables: real GDP for US, real exchange rate, real M1 for US, and US Treasury bill rate. Permanent supply and demand shocks are identified by using the same restrictions in both models. CG does not have permanent nominal component. All shocks other than demand and supply are temporary shocks in CG model.

TABLE 8

Examination of the Innovations in the Common Trend Model

Correlation coefficients

	Permanent demand	**Permanent nominal**
Countries		
CANADA	0.51	0.07
JAPAN	0.38	0.48
UNITED KINGDOM	0.35	0.13
GERMANY	0.35	0.13
FRANCE	0.52	0.66
ITALY	0.52	0.44

Notes: The correlation coefficient in the first column is the correlation between the ratio federal expenditures and the permanent demand component. The second column values are correlation coefficients between the permanent nominal trend and the Michigan Survey Data for inflation expectations. Survey data starts at 78:1, and the correlation coefficient is valid for 79:1 – 92:4.

TABLE 9

Kolmogorov-Smirnov Tests for Normality of U.S - Japanese Real Exchange Rate Responses

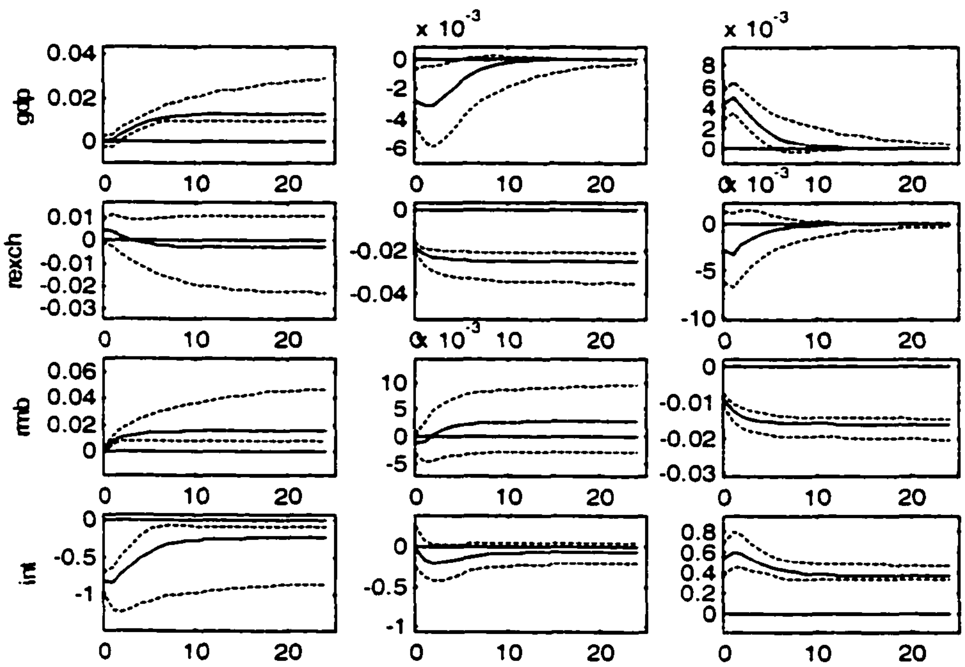
Shock	Horizon	P-Value
Supply	4	8.01×10^{-3}
	12	8.79×10^{-22}
	24	2.32×10^{-44}
Demand	4	1.78×10^{-15}
	12	2.38×10^{-19}
	24	2.35×10^{-22}
Nominal	4	4.75×10^{-22}
	12	1.28×10^{-17}
	24	0.00

Figure 1.a.

Accumulated Impulse Responses k-Quarters After Shock with 90% Confidence Intervals

Canada

Shock: Supply Demand Nominal



Japan

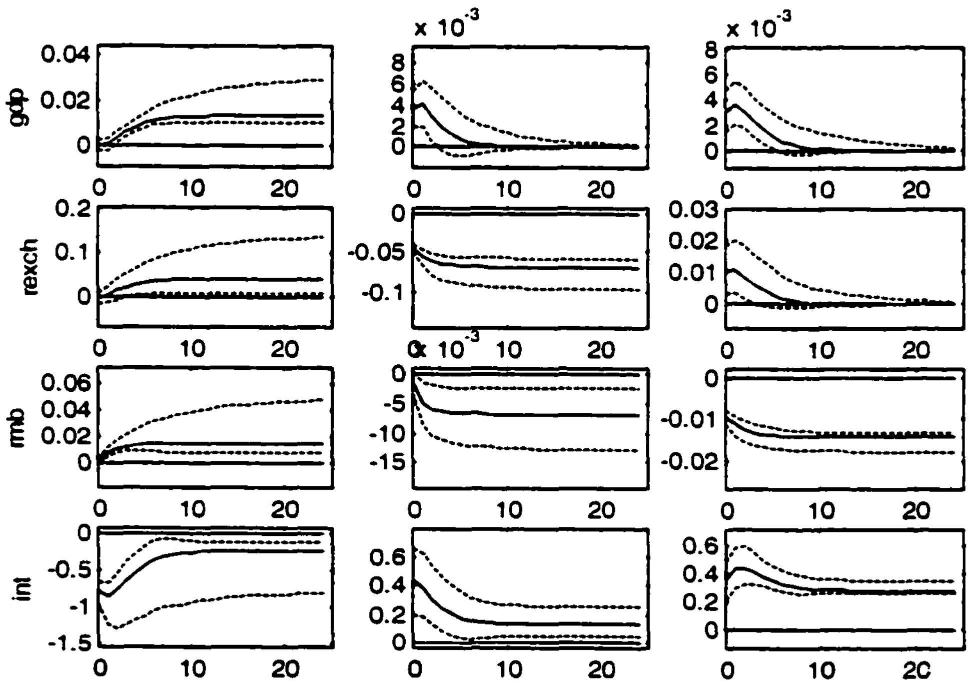
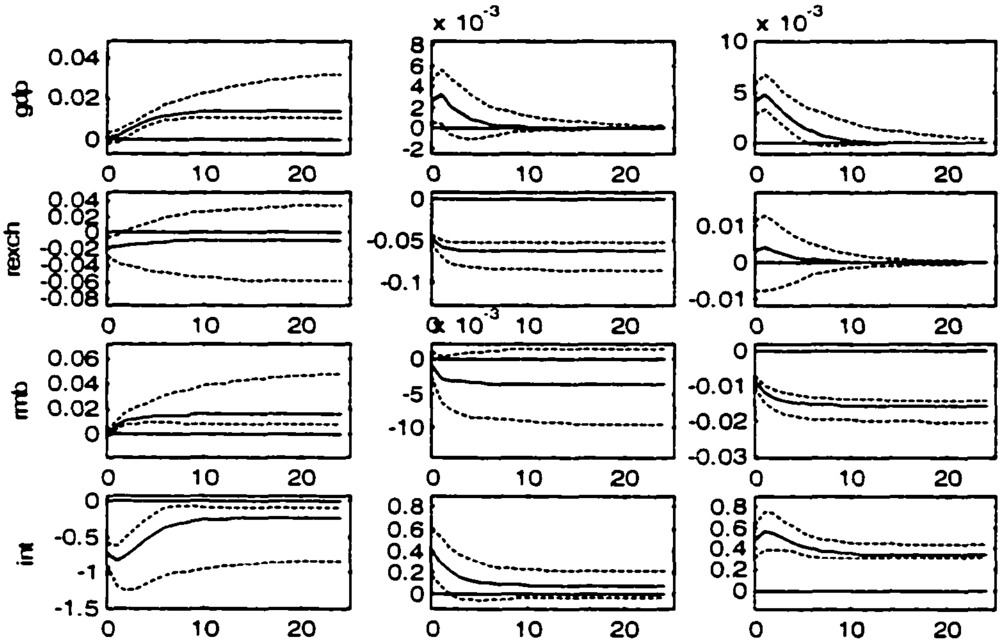


Figure 1.b.

Accumulated Impulse Responses k-Quarters After Shock with 90% Confidence Intervals

United Kingdom

Shock: Supply Demand Nominal



Germany

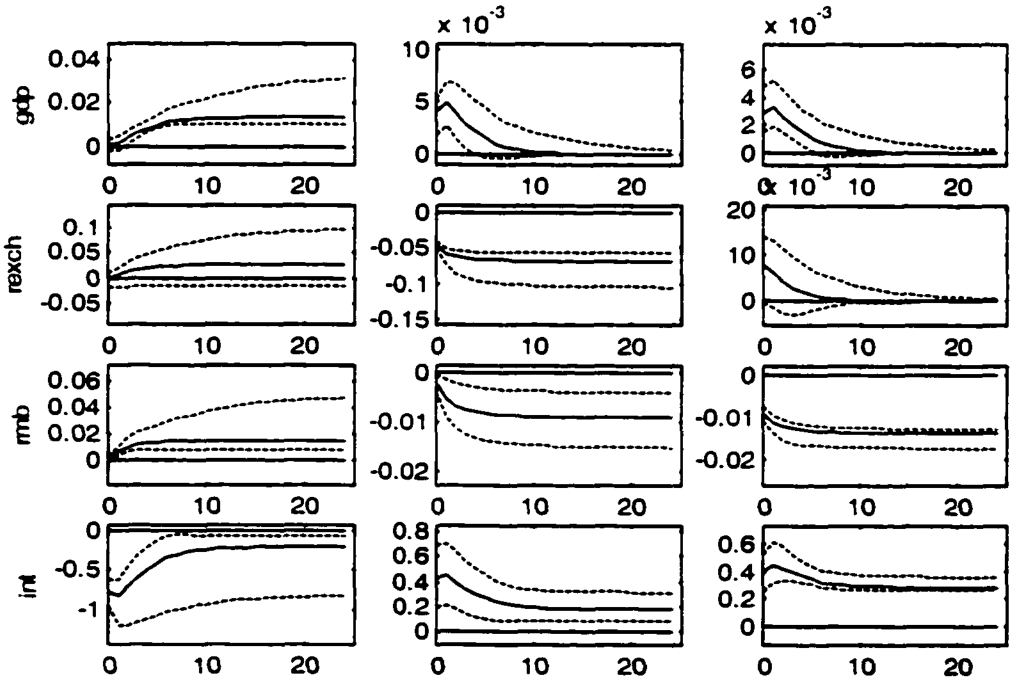
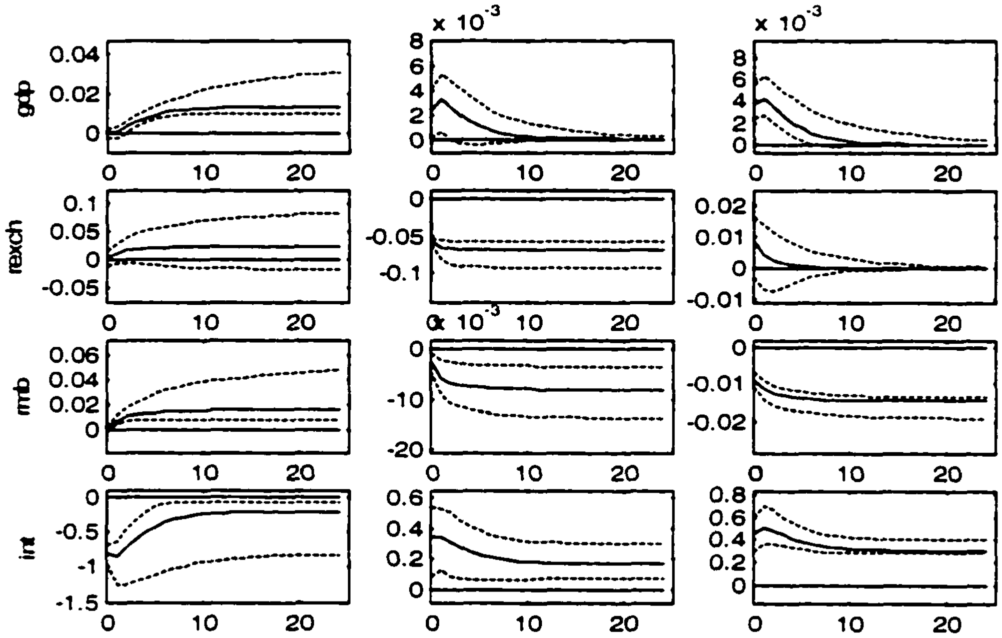


Figure 1.c.

Accumulated Impulse Responses k-Quarters After Shock with 90% Confidence Intervals

France

Shock: Supply Demand Nominal



Italy

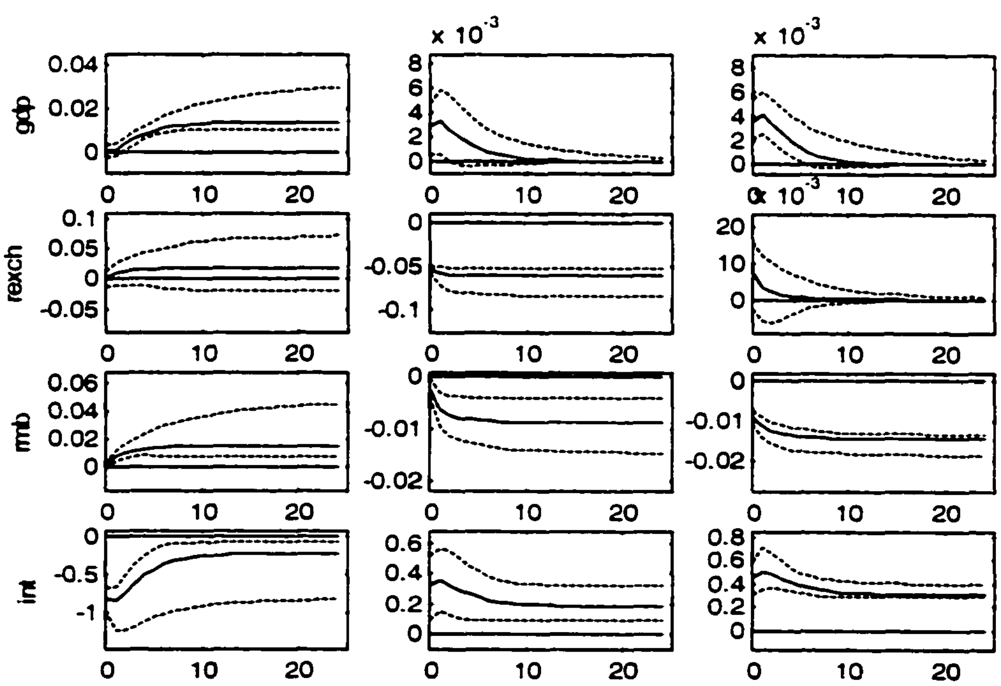
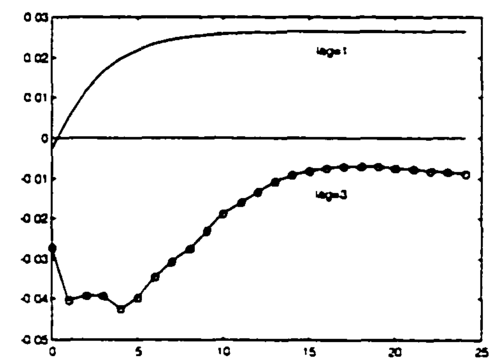


FIGURE 2

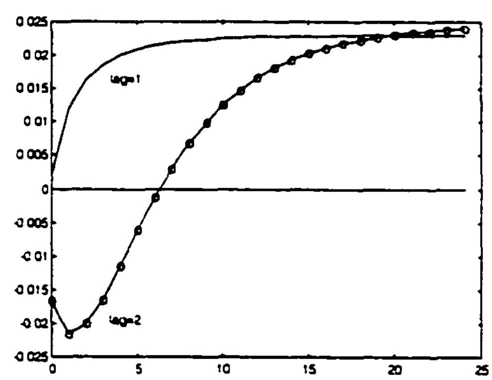
Robustness of Impulse Responses to VAR lag Specification

Cumulative Responses of Real Exchange Rate to Supply Shocks

U.S. – German Real Exchange Rate



U.S. -French Real Exchange Rate



U.S. -Japanese Real Exchange Rate

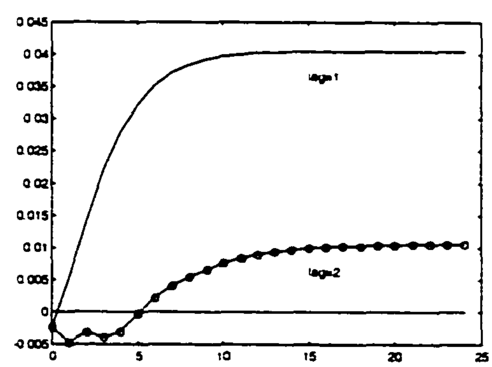
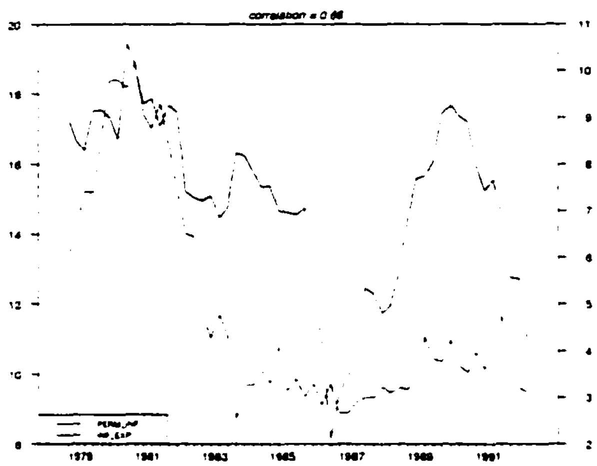


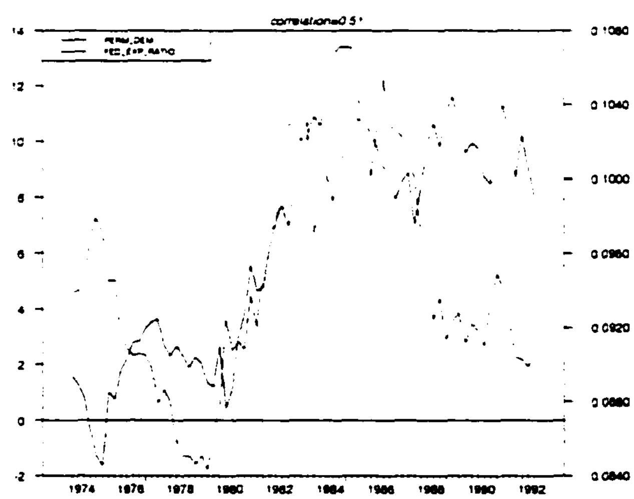
FIGURE 3

Permanent Nominal Component and Inflation Expectations from the Michigan Survey Data

FRANCE



Permanent Demand Component and Federal Spending as Ratio of GDP

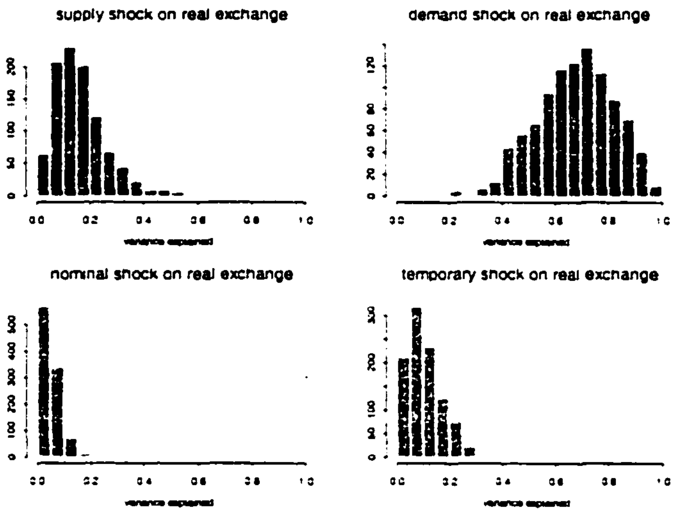


Notes: The perm_inf series is the permanent nominal component after Common Trends decomposition. The inf_exp series is inflation expectations for the next year obtained quarterly from Michigan Survey Data. The perm_dem series is the permanent demand component obtained using Common Trends approach. The fed_exp_ratio series is the ratio of federal government expenditures to output. Michigan Survey Data and data for federal expenditures are taken from FRED at the St. Louis FED Web site. Michigan Survey data starts at 78:1. The plots are on two-scale due to absence of inflation and federal expenditures as variables in the VEC model.

FIGURE 4

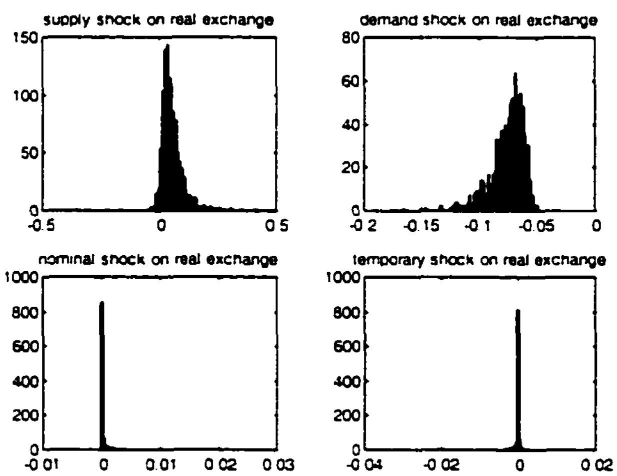
Histograms of Bootstrap Forecast Error Variance Decompositions
At 24 quarter Horizon

U.S. – Japanese Real Exchange Rate Differences



Histograms of Impulse Responses
At 24 quarter Horizon

U.S. – Japanese Real Exchange Rate



Notes: Bootstraps are from Monte-Carlo integration with 1000 repetitions. Lag length of VAR for Japan is one.

GENERAL CONCLUSIONS

Long memory in volatility of stock market returns and other financial variables has been one of the prolific areas of research in finance and economics in recent years. Regime switching processes have been proposed to mimic the persistence observed in the volatility of the series. In Chapter I, I showed through a Monte-Carlo simulation that regime switching processes may carry features of a long memory process. The long memory behaviour can be approximated by either very low transition probabilities in a Markov regime switching ARCH model or through increasing the expected waiting time in a semi-Markov process. The long memory tests I use establish a relationship between the regime switching models in a small finite system and the regime switching models where the number of switches is infinite but countable. The size and power of the MRR and GPH tests validate the additional persistence introduced by switching across different volatility regimes. The approach in Chapter I shares common features with Hsu (1997) and Liu (2000) and supports their arguments and conclusions using a simulation study.

In Chapter II, and Chapter III, I re-examine the structural modeling of real exchange rates via two separate methods. The common analysis method in the literature is to use differenced variables in the VAR system. Such an approach has its serious deficiencies, as it does not address any possible cointegration effects between the variables. Such effects are either not modeled through choice of proper non-cointegrated variables or are simply ignored. The common trends approach allow the modeler to incorporate possible cointegration effects into the structural estimation. Although common trends decomposition have found common use in other disciplines of economics, it has not yet been fully drawn up on the international finance literature, real exchange rates in particular. I use data from G-7 countries in the post-Bretton Woods period similar to the seminal paper by Clarida and Gali (1995) with the same long run restrictions imposed on the system. I identify serious cointegration effects by a choice of a slightly different data with an extended set of countries and the decompositions do not lend themselves easily to be interpreted as productivity, demand and nominal shocks. The impulse responses have incorrect signs in the long run in contrast to the common findings in the literature and Clarida and Gali (1995) model. Only for the case of Canada we have correct decomposition, that is mainly due to it being the only country satisfying no cointegration assumption in the trivariate system. I also address the significance of the impulse responses and cannot find significance for most shocks. The lag lengths selected by various criteria in the structural VAR have serious effects on the impulse responses, at times reversing even the signs in the short and the long run. I show that such perverse sign effects are mainly due to non-modeled cointegration in the VAR.

Chapter III takes the structural modeling of real exchange rates one step further by accounting for cointegration. The structural vector error correction modeling of real exchange rates is carried out within the same time frame as in Chapter II, but with slightly different variables. The four variables are real GDP, real exchange rate, real money balances, and the nominal interest rate. The money demand relationship serves as the cointegrating vector. With one cointegrating vector and four variables Chapter II identifies permanent productivity, demand and nominal shocks with the same long-run restrictions used in Chapter II. The new structural model with the cointegration effects corrects for the perverse signs in the impulse responses. The perverse sign of the real exchange rate due to a supply shock observed in common structural VAR studies is also examined and it is established that for most cases the real exchange rate depreciates in response to a positive productivity shock. The findings in Chapter III confirms my previous observation that the perverse and changing signs of the impulse responses are mainly due to sensitivity of the VAR and cointegration tests to various choice of lags. Lags that may enhance the cointegration effects lead to sign changes in the impulse responses. By comparing variance decompositions both from a structural VAR and a structural VEC model, I show that the relative importance of permanent and nominal shocks do not change much regardless of the decomposition procedure used. This serves to bolster Sarte (1997)'s argument that it is the identifying restrictions that are at the heart of differing relative importance of shocks in structural decompositions.

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